1.-Timetabling the infinite

INTRODUCTION

1 Mathematics is not usually concerned with the way the infinitely many successive steps of, for instance, a recursive $\omega-$ordered definition could in fact be carried out. It simply assumes they are carried out in their complete totalities. But the finitely or infinitely many successive steps of any definition or procedure could easily be timetabled by any sequence of instants of the same ordinality as the sequence of steps, and a one to one correspondence between both sequences. Evidently, the correspondence between instants and steps has no effect on the result of the timetabled definition or procedure. It simply states the successive instants at which each of its successive steps could take place.

2 In the next two sections we will timetabled an $\omega-$ordered sequence of definitions as a result of which we will find a new infinitist infelicity, now one of a temporal nature.

RECURSIVE DEFINITIONS

3 Let $\langle a_n \rangle_{n \in \mathbb{N}}$ be any $\omega-$ordered sequence a_1, a_2, a_3, ... and consider the following recursive definition:

$$n = 1, 2, 3, \ldots \begin{cases} n = 1: & A_1 = \{a_1\} \\ n > 1: & A_n = A_{n-1} \cup \{a_n\} \end{cases} \tag{1}$$

The result of definition (1) is assumed to be an $\omega-$ordered sequence $\langle A_n \rangle_{n\in\mathbb{N}}$ of nested sets $A_1 \subset A_2 \subset A_3 \subset \ldots$ that, according to the hypothesis of the actual infinity, exists as a complete totality.

4 Let now (t_a, t_b) be any finite interval of time and let $\langle t_n \rangle_{n\in\mathbb{N}}$ be an $\omega-$ordered and strictly increasing sequence of instants within (t_a, t_b) whose limit is just t_b, as is the case of, for example, the classic sequence defined by:

$$t_n = t_a + (t_b - t_a) \times \frac{2^n - 1}{2^n} \tag{2}$$

Definition (2) assumes time is infinitely divisible, what may, or may not, be the case in the physical world. This is not, however, an impediment to infinitist formal theories because they could be assumed to be developed in a conceptual universe in which time is arbitrarily defined as infinitely divisible.

5 Definition (1) can be timetabled by the sequence $\langle t_n \rangle_{n\in\mathbb{N}}$ in an elementary way: by assuming that each nth step takes places at the precise instant t_n. The one to one correspondence f defined by:

$$f : \langle t_i \rangle_{i\in\mathbb{N}} \longleftrightarrow \langle A_i \rangle_{i\in\mathbb{N}} \tag{3}$$

$$f(t_i) = A_i,\ \forall i \in \mathbb{N} \tag{4}$$

proves that at t_b we will have the same $\omega-$ordered totality $\langle A_n \rangle_{n\in\mathbb{N}}$ defined in (1).

A conflicting definition

6 Timetabling mathematical definitions composed of infinitely many steps reveals some significant insufficiencies on the assumed completeness of the involved $\omega-$ordered totalities. We will now examine one of them.

7 Let x and y be two natural variables (whose domain is the set of natural numbers) and consider the following $\omega-$ordered sequence of definitions:

$$\text{At each successive instant } t_n \text{ of } \langle t_n \rangle_{n\in\mathbb{N}} \begin{cases} y = 1 \\ x = n \end{cases} \tag{5}$$

where n in t_n is the same n as in $x = n$. Since t_b is the limit of $\langle t_n \rangle_{n\in\mathbb{N}}$, at t_b definition (5) will have been completed. Thus, t_b is the first instant at which the variables x and y are no longer redefined.

8 We will now prove that x and y remain well defined along the whole interval $[t_1, t_b)$. In fact, let t be any instant within $[t_1, t_b)$. Evidently, it holds $t_1 \leq t < t_b$. So, if $t = t_1$ we will have $x = 1; y = 1$. And if $t_1 < t$, there will be an index v such that $t_{v-1} < t < t_v$ because $\langle t_n \rangle_{n\in\mathbb{N}}$ is an $\omega-$ordered and strictly increasing sequence whose limit is t_b. In this case we will have $x = v - 1; y = 1$. This proves that both variables remain well defined along the whole interval $[t_1, t_b)$.

9 Since x and y remain well defined along the whole interval $[t_1, t_b)$ and no other definition takes place neither at t_b nor after t_b, both variables remain well defined at t_b and after t_b.

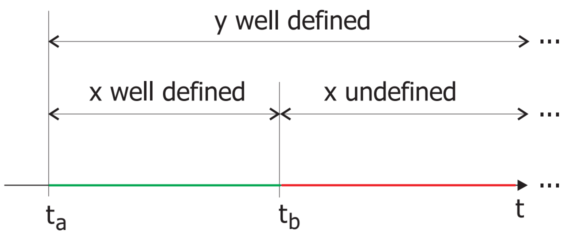

Figure 1.1: Just at t_b the natural variable x get undefined although nothing happens at t_b that could undefine x.

10 It is immediate to prove, however, that x is not defined at t_b. Although it was always defined as a natural number, its current value at t_b cannot be a natural number, otherwise that number would be the impossible last natural number or, alternatively, only a finite number of definitions would have been carried out. Notice this is not a question of indeterminacy but of impossibility: no natural number v exists such that the value of x at t_b could be v. None. So, we know nothing on the current value of x at t_b. After infinitely many correct definitions it successfully get undefined. The problem is that nothing happens at t_b that can undefine x.

11 In agreement with 9 and 10, we should conclude that, as a consequence of having being defined infinitely many successive times, at t_b the variable x is and is not defined.

12 The problem posed by the definition of x at t_b is a consequence of assuming that the $\omega-$ordered list of natural numbers exists as a complete totality despite the fact that no last number completes the list. In these conditions, x would have been successively redefined as each of the successive elements of a completed list in which no last element exist. Consequently, x would have been defined a complete totality of times in which no last definition exist. By contrast, the control variable y remains defined at t_b because it was always defined with the same value 1 and then the lack of a last definition poses (at least apparently) no problem on its current value.

Bibliography