

## 1.-Reinterpreting Riemann series theorem

### DEFINITIONS

**1** Riemann series theorem (see below) states that it is possible to reorder the summands of a conditionally convergent series in such a way that it converges to any desired number or to infinity. As we will see, the theorem only applies if infinitely many terms are involved in the rearrangement. In those conditions to converge and not to converge to any given number could be reinterpreted as a contradiction derived from the inconsistency of the actual infinity.

**2** A series  $\sum_{i=0}^{\infty} a_i$  is conditionally convergent if, and only if:

1. The series converges to a finite number  $L$ :

$$\lim_{n \rightarrow \infty} \sum_{i=0}^{\infty} a_i = L \quad (1)$$

2. The series of its positive (negative) terms diverges to positive (negative) infinite.

$$\lim_{n \rightarrow \infty} \sum_{i=0}^{\infty} |a_i| = \infty \quad (2)$$

**3** Riemann series theorem states that by the appropriate rearrangement of its terms, any conditionally convergent series may be made to converge to any given finite number or to infinity.

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DISCUSSION

**4** We will exclusively deal with conditionally convergent series of real numbers that may converge to different finite numbers by rearrangements based on the application of the commutative, associative and distributive properties of the elementary arithmetic operations in the field of real numbers.

**5** Let  $S = \sum_{i=0}^{\infty} a_i$  be any conditional convergent series and  $v$  any natural number. Consider the sum of the first  $v$  terms of  $S$  and denote it by  $S_v$ . Since the number  $v$  of summands is finite, the number of its possible permutations is also finite. Let  $S_{v,1}, S_{v,2}, S_{v,3}, \dots$  be the sum of successive rearrangements of  $S_v$ , each of them accomplished after a finite number of applications of the associative, commutative and distributive property to the previous rearrangement  $S_{v,i-1}$ . In consequence, if for a certain  $i$  we would have:

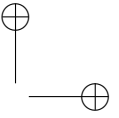
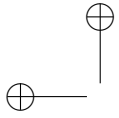
$$S_{v,i-1} \neq S_{v,i} \tag{3}$$

we would have to conclude that after a finite number, say  $k$ , of operand reorderings, the result of the sum changes. But since  $k$  is finite the global reordering can be resolved in a finite sequence of single reorderings, resulting each one of them from a single application of the commutative, associative or distributive properties. So, at least one of those single applications must be involved in (3). But this impossible because no simple application of the commutative, associative or distributive properties can alter the result of a sum if those properties hold as they must hold in the field of the real numbers. Inequality 3 is then impossible for any finite number.

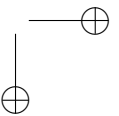
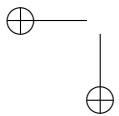
**6** We can therefore confirm that for any  $v$  in  $\mathbb{N}$  the sum of first  $v$  terms of any conditionally convergent series is always the same, and then that the sum does not depend upon the operand rearrangement. Only when the number of summands is infinite the sum depends upon the summand rearrangement. We must conclude it is the infinite number of summands the cause of that dependence.

**7** According to Riemann series theorem, if  $S$  is any conditionally

convergent series and  $r$  any real number the sum of its infinitely many terms is and is not equal to  $r$ , depending on the order the terms of the series are summed. This is the type of result you can expect if the hypothesis of the actual infinity were inconsistent. Riemann series theorem could therefore be reinterpreted as a proof of the inconsistency of the actual infinity hypothesis. And that possibility, as legitimate as any other, should be explicitly declared in the theorem statement.



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## Bibliography