COEVOLUTION. NEW THERMODYNAMICS THEOREMS

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(Revised version)

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ABSTRACT. The formal consideration of the concept of interaction in thermodynamic analysis makes it possible to deduce, in the broadest terms, new results related to the coevolution of interacting systems, irrespective of their distance from thermodynamic equilibrium. I prove here the existence of privileged coevolution trajectories characterized by the asymmetrical way in which systems progress along them. Coevolution along these trajectories produce the minimal amount of entropy within the evolving systems.

1. INTRODUCTION

One of the primary objectives of non equilibrium thermodynamics is the analysis of open systems, systems which -like biological systems- maintain a continuous exchange (flow) of matter and energy with their environment. Through these flows, open systems organize themselves in space and time. However, the behaviour of these systems differs considerably according to whether they are close or far from equilibrium. Close to equilibrium, the phenomenological relationships which bind the flows to the conjugate forces responsible for them are roughly linear, i. e. of the type:

$$J_i = \sum_j L_{ij} X_j,\ i, j = 1, 2, \ldots, n \tag{1}$$

where J_i are the flows, X_j the generalized forces, and L_{ij} are the so called phenomenological coefficients giving the Onsager Reciprocal Relations [3, 4]:

$$L_{ij} = L_{ji},\ i \neq j \tag{2}$$

In these conditions, Prigogine's Theorem [5] asserts the existence of steady states characterized by minimum entropy production. Systems can assimilate their own fluctuations and be self-sustaining.

Far from equilibrium, on the other hand, the phenomenology may become clearly non-linear allowing the development of certain fluctuations that will reconfigure the system [1]. Thermodynamics of irreversible processes can now only provide information regarding system stability. But there is no physical potential driving the system evolution [6]. The objective of the following discussion is just to derive certain formal conclusions related to that evolution in the case of interacting open systems evolving far from equilibrium.

2. Definitions

The following discussion is based on two prior assumptions:

(1) There exist open systems whose available resources are limited.
(2) Owing to these limitations, such systems compete with one another to maintain their matter and energy flows.

Both assumptions lead directly to the concept of interaction. But before proposing a formal definition of interaction let us examine Figure 1 in order to obtain an intuitive idea of the type of problem being dealt with. Figure 1 provides a schematic representation of three open systems away from thermodynamic equilibrium. In the case (A) the system if not subjected to interaction with other systems, and maintains a through-flow of matter and energy which depends solely -without going into phenomenological details- on its own degree of imbalance. In the case (B) the systems interact with each other, and their respective flows depend on the degree of imbalance of both systems simultaneously. It is this situation which will be explored here as broadly as possible, particularly from the point of view of the coevolution of both systems.

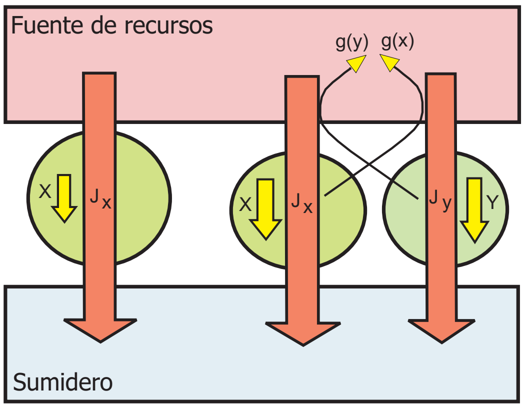

FIGURE 1. *Circles represent open systems far from thermodynamic equilibrium. Rectangles represent significant parts of their environment. Matter and energy flows are shown as large arrows crossing systems from sources to sinks. Yellow vertical arrows X and Y represent the generalized forces responsible for the flows. Curved arrows g(X) and g(Y) represent the interactions between the systems. In the absence of interaction with other systems, the flow depends only on the intensity of the own force that maintain the system away from equilibrium ($J_X = f(X)$). When two systems compete with one another in order to maintain their flows, these flows depend on the intensity of the generalized forces (degree of imbalance) in both systems simultaneously. In these conditions, the flows are expressed by expression of the type $J_X = f(X) - g(Y)$; $J_Y = f(Y) - g(X)$ (see text).*

Let f and g be any two type $\mathbb{C}^2$ functions (continuous functions with first and second derivatives which are also continuous functions) defined on the set $\mathbb{R}$ of real numbers and such as:

$$f \text{ and } g \text{ are strictly increasing, and } f(0) = g(0) = 0 \quad (3)$$

$$(f - g) \text{ is strictly increasing} \quad (4)$$

$$\text{and its first derivative } (f - g)' \text{ is also increasing.} \quad (5)$$

Function f will be referred to as the phenomenological function and g as the interaction function. The flow in a system will be given by $J_X = f(X)$ where X is the generalized force, a measure of the degree of the system's imbalance or distance from equilibrium. In consequence $X \geq 0$ ($X = 0$ at equilibrium). X will also be used to designate the system. Let us justify the constraints on f and g. Firstly, $f(0) = g(0) = 0$ indicates that in the absence of generalized forces (thermodynamic equilibrium) there is neither flows nor interactions. The increasing nature of both functions indicate that the intensity of flows and interactions increase with the generalized forces, or in other words, as we move away from equilibrium. Constraints (4) and (5) imply that systems are progressively as sensitive to their own forces as they are to the forces of the other interacting system, or more so.

Given two systems with the same phenomenology f and forces X and Y respectively, an interaction can be said to exist between them if their flows can be described by the following expressions:

$$J_X = f(X) + C_{XY} g(Y), \; -1 \leq C_{XY} \leq 1 \quad (6)$$

$$J_Y = f(Y) + C_{YX} g(X), \; -1 \leq C_{YX} \leq 1 \quad (7)$$

where C_{XY} and C_{YX} are the interaction coefficients. We shall deal here only with the double negative interaction, assuming -while still speaking in general terms- that $C_{XY} = C_{YX} = -1$. Or in other words, that:

$$\begin{cases} J_X = f(X) - g(Y) \\ J_Y = f(Y) - g(X) \end{cases} \quad (8)$$

In addition, the only (X, Y) pairs permitted will be those which give a positive value for expressions 8 above. (X, Y) pairs giving $f(X) - g(Y) = 0$ shall be termed the points of extinction of system X. The same applies to system Y. The interaction thus defined may be seen as an external constraint imposed by the systems on each other, forcing them to reach steady states simultaneously.

3. Entropy production

The purpose of the following discussion is to obtain information relating to the coevolution of two open systems, X and Y, which interact with each other. As is well known, the entropy balance for an open system can be expressed as:

$$dS = d_i S + d_e S \quad (9)$$

where d_iS represents the entropy production within the system due to flows, and d_eS is the entropy exchange with its surroundings. The second law of thermodynamics dictates that always $d_iS \geq 0$ (zero only at equilibrium).

One of the most interesting aspects of Thermodynamics of Irreversible Processes is the inclusion of time in its equations:

$$\frac{d_iS}{dt} = \sum_j X_j J_j \tag{10}$$

In our case, for system X we have:

$$\frac{d_{iX}S}{dt} = X[f(X) - g(Y)] \geq 0 \tag{11}$$

and for system Y:

$$\frac{d_{iY}S}{dt} = Y[f(Y) - g(X)] \geq 0 \tag{12}$$

and the joint production of internal entropy:

$$\frac{d_{iX}S}{dt} + \frac{d_{iY}S}{dt} = \dot{S}(X,Y) = X[f(X) - g(Y)] + Y[f(Y) - g(X)] \tag{13}$$

Graphically, $\dot{S}(X,Y)$ is a surface in the space defined by the axes $\dot{S}, X$ and Y (Figure 2). A curve in the plane XY represents a possible coevolutionary history for both systems (as long as X and Y satisfy (11) and (12). Its projection on $\dot{S}(X,Y)$ shows the cost of this history in terms of internal entropy production.
The analysis of the surface $\dot{S}(X,Y)$ is thus of great interest. We will examine it by analyzing its intersections with planes parallel to the plane $\dot{S}X$. Or in other words, by analyzing the joint production of internal entropy for each Y constant. For example, if $Y = b$ we will have:

$$\dot{S}(X,b) = X[f(X) - g(b)] + b[f(b) - g(X)] \tag{14}$$

If we derive (14) with respect to X, calling this derivative $h(X)_b$, we will have:

$$\frac{\partial \dot{S}(X,b)}{\partial X} = h(X)_b = f(X) - g(b) + Xf'(X) - bg'(X) \tag{15}$$

where, according to restrictions (3), (4) and (5), $h(X)_b$ is strictly increasing. Moreover:

$$h(0)_b = -g(b) - bg'(b) < 0 \tag{16}$$

and:

$$h(b)_b = f(b) - g(b) + b[f'(b) - g'(b)] > 0 \tag{17}$$

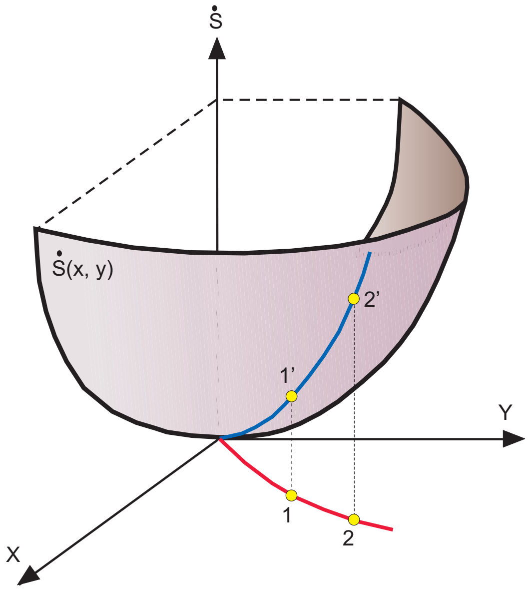

FIGURE 2. *Curve 1-2 represents any coevolutionary history between system X and Y. Curve 1'-2', its projection on* $\dot{S}(X,Y)$*, represents the joint production of internal entropy throughout that history.*

According to Bolzano's theorem, therefore, there must exist a single point X_b at which:

$$h(X_b)_b = 0,\ 0 < X_b < b \tag{18}$$

Furthermore, since $h(X)_b$ is strictly increasing we have:

$$\frac{\partial^2 \dot{S}(X,b)}{\partial X^2} = h'(X)_b > 0 \tag{19}$$

X_b is therefore a minimum of $\dot{S}(X,b)$ (Figure 3)

We have shown that for each Y constant there exists a single value of X which minimizes the joint production of internal entropy. The (X,Y) pairs which satisfy this conditions form a curve which will henceforth be termed the coevolution trajectory. Its equation is:

$$f(X) - g(Y) + Xf'(X) - Yg'(X) = 0 \tag{20}$$

For $Y = 0$ we have:

$$f(X) + Xf'(X) = 0 \tag{21}$$

which is only true for $X = 0$. The point (0, 0) thus belong to the trajectory. All its points thus give:

$$0 \leq X < Y \tag{22}$$

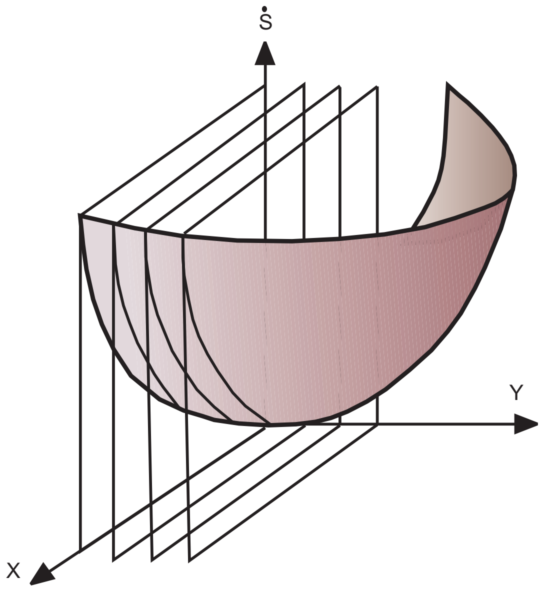

FIGURE 3. *Intersection of surface $\dot{S}(X,Y)$ with plane $Y = b$ parallel to $\dot{S}X$. The point X_b is a minimum of $\dot{S}(X,b)$.*

Further information can still be obtained regarding this. In fact, deriving (20) with respect to X we obtain:

$$2f'(X) + Xf"(X) - g'(Y)Y' - g'(X)Y' - Yg"(X) = 0 \tag{23}$$

whence:

$$Y' = \frac{2f'(X) + xf"(X) - Yg"(X)}{g'(X) + g'(Y)} \tag{24}$$

$$= \frac{h'(X)_Y}{g'(X) + g'(Y)} > 0 \tag{25}$$

We are therefore dealing with a strictly increasing curve, contained in the region defined by the axis Y and the bisector $Y = X$. This means that system Y is always ahead of system X with respect to equilibrium. A point of extinction may exist for system X if the rate of increase of Y with respect to X exceeds the rate of increase of $(f - g)$

The same reasoning cam be applied to the sections of $\dot{S}(X,Y)$ parallel to $\dot{S}Y$. As consequence, a new coevolutionary trajectory will be obtained. Both trajectories are symmetrical with respect to the bisector $Y = X$, sharing the point (0, 0) of thermodynamic equilibrium. All this allows us to plot the surface $\dot{S}(X,Y)$ as shown in Figure 4, and reflect the results obtained so far in the form of the following theorems:

Theorem 1. *For each pair of systems interacting with one another, two alternative and symmetrical coevolution trajectories exist, formed by a succession of steady states characterized by the minimum joint entropy production within the systems.*

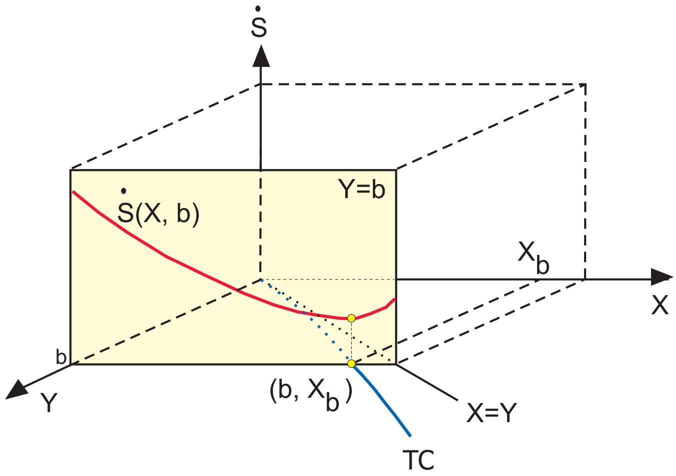

FIGURE 4. $\dot{S}(X,Y)$ *plotted in four sections orthogonal two by two. Curves ct represent the coevolution trajectories*

Theorem 2. *(St Matthew's Theorem) Systems progressing along coevolutionary trajectories do so in such a way that one of them is always ahead of the other with respect to thermodynamic equilibrium.*

4. GENERALIZATION

Our analysis has been restricted to the double negative interaction (-, -), but the same method can be applied to other types of interaction (+, +), (+, 0), (+, -), (-, 0), where "+" indicates positive interaction and "0" the absence of interaction. Though it is not our intention to analyze these here, we will prove the following:

Theorem 3. *In order for coevolution trajectories to exist, at least one of the interactions must of necessity be negative.*

In fact, for the case (+, +) instead of (8) we have

$$\begin{cases} J_X = f(X) + g(Y) \\ J_Y = f(Y) + g(X) \end{cases} \tag{26}$$

and instead of (13):

$$\dot{S}(X,Y) = X[f(X) + g(Y)] + Y[f(Y) + g(X)] \tag{27}$$

whence:

$$\frac{\partial \dot{S}(X,b)}{\partial X} = f(X) + g(b) + Xf'(X) + bg'(X) \tag{28}$$

$$\frac{\partial \dot{S}(a,Y)}{\partial Y} = f(Y) + g(a) + Yf'(Y) + ag'(Y) \tag{29}$$

But both derivatives are always positive and cancel out simultaneously and exclusively at the point $(0, 0)$ of equilibrium. There are therefore no critical points for $\dot{S}(X, b), \dot{S}(a, Y)$, which in accordance with (28) and (29) are strictly increasing. The same results are obtained for (0, +) and (+, 0), which proves theorem 3.

On the other hand, if we consider n systems instead of 2, the following expressions would be obtained for flows:

$$J_{X_i} = f(X_i) + \sum_j C_{ij} g(Xj), \ -1 \leq C_{ij} \leq 1, \ i, j = 1, 2, \ldots n \tag{30}$$

where C_{ij} represents (as the binary case) the type and degree of interaction between system X_i and X_j. The joint production of internal entropy could thus be expressed as:

$$\dot{S}(X_1, X_2, \ldots X_n) = \sum_i X_i \left[f(X_i) + \sum_j C_{ij} g(X_j) \right] \tag{31}$$

$\dot{S}(X_1, X_2, \ldots X_n)$ is a hypersurface on $\mathbb{R}^{n+1}$ in which we can consider $\sum(n-1)$ three-dimensional subspaces $\dot{S}X_iX_j$ in order to analyze the interaction between the system X_i and the system X_j using the same method.

5. Discussion

The most relevant feature of an open system is its exchange boundary, and particularly the set of processes which arise and are maintained at the expense of matter and energy flows. These processes provide the system with a certain degree of organization, i. e. a given space-time configuration or arrangement from which then maintenance of the flows is derived. But not all processes are equally efficient in flows maintenance. Most efficient processes have a greater capacity for assimilating their own fluctuations, and thus a greater chance of being self-sustaining. To a certain extent, the degree of organization of an open system may be interpreted as a measure of its ability to maintain the flows.

We have shown that coevolution trajectories are the only histories of coevolution compatible with a maximum level of organization of systems; in other words, they are histories of maximum efficiency in flows maintenance. Simply in view of the imposition of external constraints like mutability and selective pressure suffered by certain types of systems (biological, economic, etc.), it is evident that these trajectories are to be taken as true references in the coevolution of these type of systems. In addition to the existence of the trajectories, perhaps the most striking result is their progression along them referred to in theorem 2. There is compatibility with the empirical data inspiring what many biologist have referred to as St Matthew principle [2]

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