1.-Relativity of a visible laser beam

Are not the Rays of Light very small Bodies emitted from shining Substances?

Isaac Newton

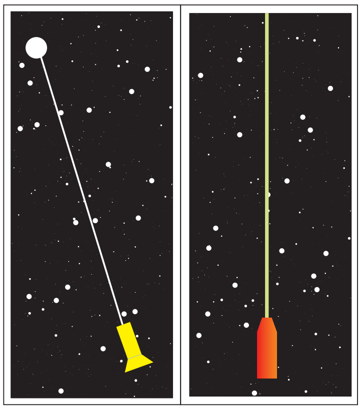

Figure 1.1: Left: Inferred photon trajectory between the observed object and the observing device. Right: A visible laser beam

Introduction

1 For obvious reasons, photons star in most of the discussions on the special theory of relativity. Typically, they are assumed to move between two points and then the corresponding trajectories are analyzed from the perspective of two or more different inertial reference frames in relative motion. Since the trajectories are not visible, they are represented by straight lines that connect the start point with the end point of the path the photon is assumed to follow. In this sense, a visible laser beam provides a fundamental advantage since the entire trajectory is visible.

2 The discussion that follows is a thought experiment that makes use of a visible laser beam to pose the following problem: RF_o is the

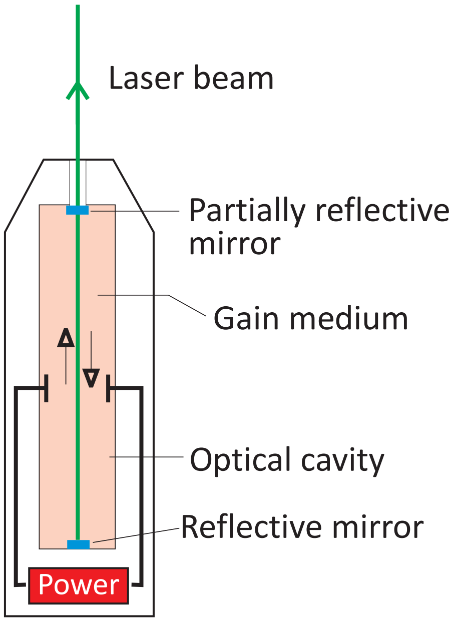

Figure 1.2: Schematic depiction of a laser source [1]. Only those photons that reflect in the direction perpendicular to the mirrors will be released in the laser beam [1].

proper inertial frame of a laser source S that emits a visible laser beam in the vertical direction, i.e. in a direction orthogonal to the X_o axis of RF_o, will this laser beam be seen as a moving vertical line in other inertial frame that moves relative to RF_o in the direction parallel to X_o?. As we will see, the answer put into question the analog interpretation of the special theory of relativity.

Discussion

3 As is well known, the relativistic aberration of light results from the relative motion between the observer and the observed source of light. In the case of a star, for instance, the main consequence is its apparent displacement in the direction of the relative motion. The relativistic aberration we will examine in this section has also to do with the relative motion of the light's source, although in this case it will be the source of a visible laser beam.

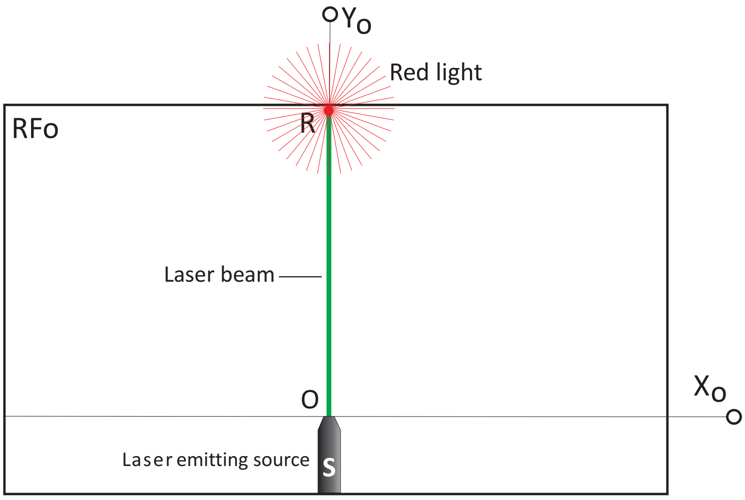

Figure 1.3: The visible laser beam in the proper reference frame RF_o of its emitting source S.

4 Let RF_o be the lab where a laser source S emits a visible vertical laser beam whose trajectory coincides with the Y_o axis of RF_o, being its starting point at the origin O of the frame RF_o. The beam impacts at point R of the lab's ceiling where a light sensor emits a red light when it is activated by the laser (Figure 1.3). Being a theoretical discussion, the emitting source will be assumed to be sufficiently large as to make it possible the relativistic observations involved in the discussion.

5 The starting point O of the laser beam and the point R of the lab's ceiling where the light sensor is placed, are, therefore, two fixed points of RF_o where two specific facts take place: the emitting of the laser and the emitting of the red light.

6 We will begin by proving that, according to the first principle of the special theory of relativity, in the frame RF_o the laser beam can only

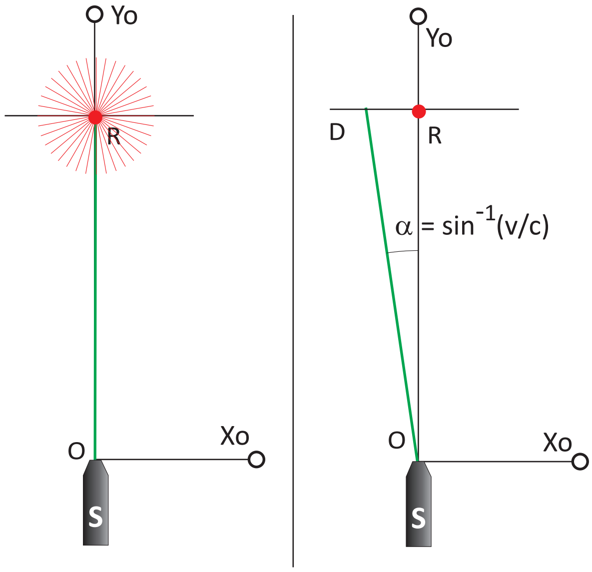

Figure 1.4: In RF_o the laser beam can only follow the trajectory OR (left), otherwise it would be possible that RF_o observers calculate the (absolute) velocity of its proper reference frame (right)

be seen as a vertical straight line segment whose endpoints are O and R. In effect, consider a photon of the laser beam and assume that after a time t_o from its emission at point O it impacts at a ceilling's point D different from R. Since it was emitted in the direction from O to R the impact on D can only means the lab moved a distance DR during the time t_o in the direction from D to R. So, DR could be used to calculate the absolute velocity u of RF_o (Figure 1.4):

$$u = \frac{DR}{t_o} \tag{1}$$

which goes against a well known derivation of the first principle of the special theory of relativity that states the impossibility of absolute reference frames with respect to which absolute velocities could be determined. Consequently, in RF_o the laser beam can only be seen as a vertical straight line segment whose endpoints are O and R.

7 We have just proved that in the frame RF_o the laser beam impacts at the ceiling's point R. In consequence the light sensor will be activated and will emit a visible red light. This red light will be seen for all observers, be them or not in relative motion with respect to RF_o.

8 It is interesting to note that, if in the place of a photon, S fires a massive bullet in the same vertical direction, the bullet would also impact at the same point R. Thus, from the point of view of the RF_o observers the photons of the laser beam behaves as a material object mechanically linked to the lab, an issue that we have already discussed in the previous chapter and that will be continued in the next one.

9 Let us now examine the laser beam from the perspective of an inertial reference frame RF_v from which RF_o moves at a velocity v in the X_o-direction, from left to right. As in all discussions in this book[1], we will assume the origins of the spacetime diagrams of both frames coincides at instant 0 when the laser is fired.

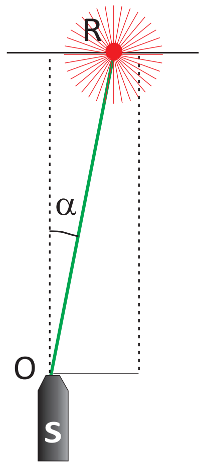

Figure 1.5: O and R should be seen vertically aligned in all reference frames that moves in the direction of X_o because Lorentz transformation cannot explain $\alpha > 0$.

[1]Se Chapter on conventions.

10 From the perspective of RF_v, each laser beam photon begins to move at point O, where it is emitted by S, and ends at point R where the light sensor is activated. The points O and R, which are mechanically linked to the lab RF_o, are on a vertical line in RF_o, and so they must be in RF_v. Indeed, assume O and R are not the endpoints of a vertical line in RF_v, being the point R displaced, for instance, to the right with respect to the point O. We would have to conclude that relative motion makes the lab appears deformed in such a way that the ceiling is displaced to the right with respect to the floor.

11 But such a deformation would be incompatible with Lorentz transformation: Since the clocks at O and R are synchronized for both RF_o and RF_v observers,[2] if (x_{oO}, t_o), (x_{oR}, t_o) are the proper space-time coordinates of O and R at any instant t_o in RF_o, the corresponding coordinates in RF_v will be:

$$x_{vO} = \gamma(x_{oO} + vt_o) \tag{2}$$

$$x_{vR} = \gamma(x_{oR} + vt_o) \tag{3}$$

And since $x_{oO} = x_{oR}$, we will have:

$$x_{vR} - x_{vO} = \gamma((x_{oR} + vt_o) - (x_{oO} + vt_o)) = 0 \tag{4}$$

which means that x_{vO} and x_{vR} are on a vertical straight line parallel to Y_v. In consequence OR is a vertical line in RF_v and then the laser beam is seen as a moving vertical line from the perspective of this frame.

12 We will now prove the impossibility for the laser beam to be seen as a moving vertical line in RF_v. But let us before recall the classical example of a photon that move vertically between two horizontal reflecting mirrors. In the proper frame of the mirrors the photon follows vertical trajectories, whereas in other frame RF_v from which RF_o

[2]They are not separated in the direction of the relative motion

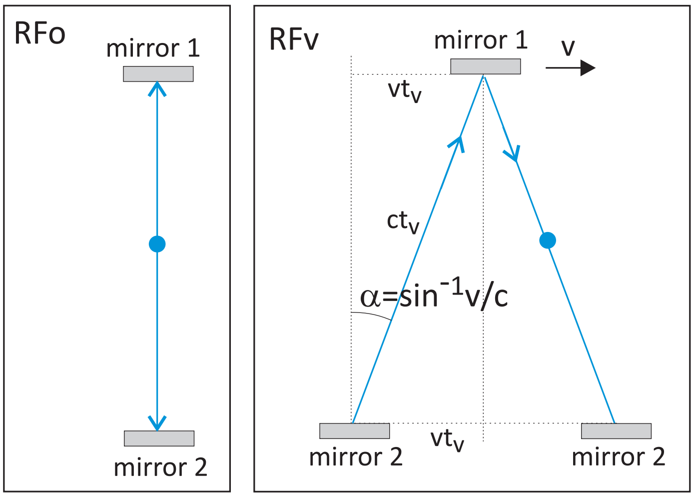

Figure 1.6: The classical example of a photon reflected vertically by two horizontal mirrors in the proper frame of the mirrors (left) and in a frame RF_v from which RF_o moves from left to right (right).

moves from left to right in the X_o direction, it follows trajectories inclined from the vertical by an angle $sin^{-1}v/c$ (Figure 1.6). As we know, the dilatation of time with relative motion derives from these different trajectories.

13 For RF_v observers, the photons in the optical cavity of the emitting source S that are released in the laser beam are not those that reflect in the the direction perpendicular to the mirrors but those that reflect in a direction α such that:

$$\alpha = sin^{-1}\frac{vt_v}{ct_v} = sin^{-1}\frac{v}{c} \tag{5}$$

where t_v is the time it takes a photon to go from a mirror to the other. Accordingly, in RF_v the laser beam is a line inclined from the vertical by an angle $sin^{-1}v/c$.

14 Now then, if the laser beam is not vertical in RF_v then it cannot impact at point R, otherwise the points O and R would not be ver-

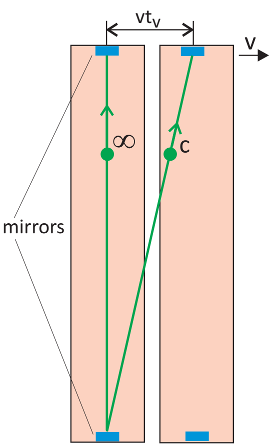

Figure 1.8: From the perspective of RF_v to traverse the distance

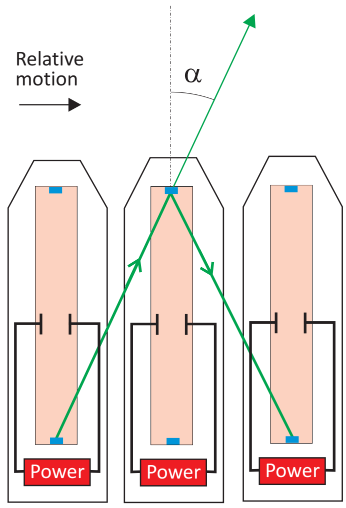

Figure 1.7: Successive reflections of photons inside the optical cavity of the laser source according to RF_v observes.

tically aligned, which is impossible according to 11. In consequence, and from the perspective of RF_v observers, the light sensor R is not activated and the red light will not be emitted. Notice this consequence (the light sensor is not fired) holds for all possible non vertical trajectories of the laser beam.

Conclusions

15 We have come to a formally inconsistent conclusion, the light sensor at point R is and is not activated. Therefore, at least one of the above arguments has to be wrong. Let us analyze the corresponding consequences of each alternative.

16 If the wrong argument is that of the RF_o observers then there will exist a reference frame with respect to which those observers will be able to measure the velocity of its own frame, which according to the first Principle of Relativity is impossible.

17 If the observers in RF_v are wrong then the photons released in the laser beam can only be those that reflect in the direction perpendicular to the mirrors (14). In consequence, an taking into account the relative velocity is greater than zero and parallel to the mirrors, those photons would have to move instantaneously from a mirror to the other, i.e. at an infinity speed, which goes against the second Principle of Relativity.

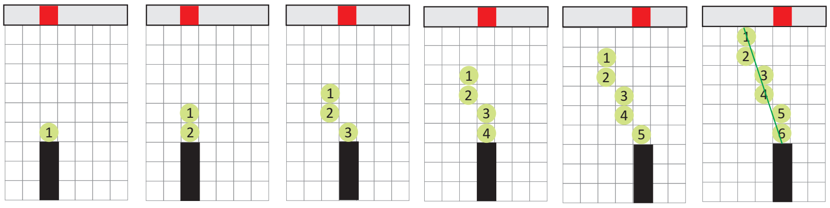

Figure 1.9: Once released, each photon lasts a time greater than zero to reach the ceiling of RF_o. During that time and from the point of view of RF_v observers, the laser source moves towards the right a distance greater than zero. In consequence, from the perspective of RF_v, the laser beam cannot be vertical.

18 The problem behind the above contradiction[3] is the impossibility to refer the motion of the photons to a frame independent of RF_o and of RF_v, which in turn is a consequence of the way the first Principle of Relativity states the universality of physical laws: by making reference to reference frames.

19 Since the points of the continuum spacetime lack of physical meaning, motion has to be referred to arbitrary reference frames. This servitude does not exist in discrete spacetimes, where motion can be (at least theoretically) described in terms of particular sets of sits that are the same in all reference frames.

20 In these conditions the motion of a photon is really independent

[3]From RF_v the laser beam is seen as a moving vertical line that cannot be vertical.

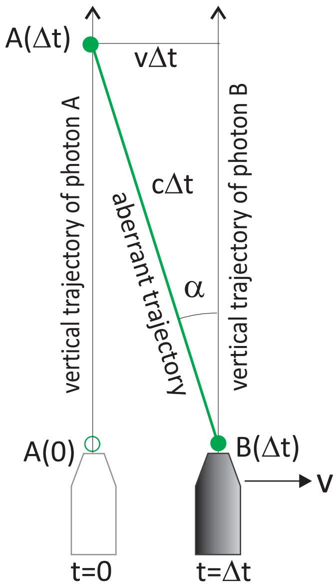

Figure 1.10: Vertical trajectory of two photons *a* and *b* at two instants 0 and Δt, and the resulting aberrant trajectory due to the relative motion of the light source.

of the relative motion of its emitting source. Although the relative motion of the source may originate an apparent or aberrant trajectory (Figure 1.10).

Bibliography

[1] Benjamín Alonso Hernández et alter, *El láser. la luz de nuestro tiempo*, Centro de Láseres Pulsados Ultracortos Ultraintensos. Universidad de Salamanca, 2010.