1.-Relativity in CALM

> . . . the very notion of a point in spacetime is incorrect
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INTRODUCTION

1 The special theory of relativity is usually introduced as a revolutionary theory on space and time. Indeed, the theory claims that space and time are intimately entangled in a four dimensional continuum. However, the fine structure of spacetime continues to be a densely ordered continuum. i.e. a classical continuum: between any two of its points an uncountable infinitude of other different points do exist. It is therefore an infinitist theory on space and time, and then a theory whose formal consistency depends upon the formal consistency of the actual infinity hypothesis. It is surprising how little attention physicists pay to this fundamental question.

2 If the hypothesis of the actual infinity were inconsistent then all imaginable continuums would also be inconsistent. And, if reality is itself consistent, space and time could only be of a discrete nature. In those conditions, two questions inevitably arise: would the special theory of relativity be affected by spacetime discreteness? And in the opposite direction: could the special theory of relativity serve to confirm the discrete nature of space and time? By way of illustration, the short discussion that follows reinterprets Lorentz transformation in discrete terms. The remainder of the book explores some relativistic conflicts that could be indicating the discrete nature of space and time.

3 Let us consider a CALM in which Pythagoras d-theorem holds. Assume we need to transform the length of the sides of a right angled triangle in this geometry into the corresponding lengths in the continuum geometry, and viceversa. Let h be the number of sits of the hypotenuse and x and y the number of sits of the legs, being $x \leq y$. In the discrete geometry of a CALM we will have $h = y$.

4 If σ is the length of a sit in the continuum geometry, then in this geometry the length of the legs will be $x\sigma$ and $y\sigma$, but the length of the hypotenuse will not be $h\sigma = y\sigma$ because, according to classical Pythagoras theorem, the length of the hypotenuse is greater than the length of both legs.

5 Let then $h'\sigma$ be the length of the hypotenuse in the continuum model. We can write (classical Pythagoras theorem):

$$(h'\sigma)^2 = (y\sigma)^2 + (x\sigma)^2 \tag{1}$$

Or:

$$(y\sigma)^2 = (h'\sigma)^2 - (x\sigma)^2 \tag{2}$$

and then:

$$y = \sqrt{h'^2 - x^2} \tag{3}$$

6 On the other hand, the ratio between the continuum and the discrete hypotenuses will be:

$$\frac{h'\sigma}{h\sigma} = \frac{h'}{h} \tag{4}$$

$$= \frac{h'}{y} \tag{5}$$

$$= \frac{h'}{\sqrt{h'^2 - x^2}} \tag{6}$$

$$= \frac{1}{\sqrt{1 - x^2/h'^2}} \tag{7}$$

Note the ratio h'/h has the form of the relativistic factor γ.

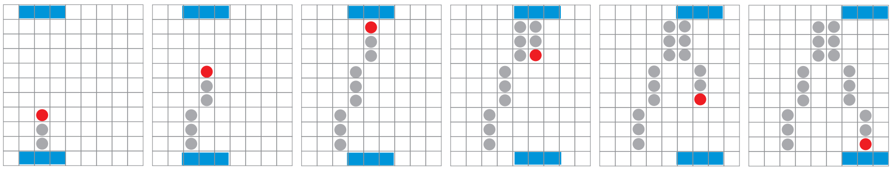

Figure 1.1: Digital Michelson-Morley: A photon moving in the direction perpendicular to v also traverses $2y$ sits (2×9 in the Figure).

7 Following the same example, and for illustrative purposes only, consider a Michelson-Morley-like interferometer moving in a bidimensional CALM with uniform linear motion of velocity v. If s is the number of sits of its arms, along the arm parallel to v light traverses $(s + x)$ sits when it moves in the same direction as the arm, and $(s - x)$ sits when it moves in the opposite direction, being x the number of sits the apparatus moves (parallel to this arm) while light traverses the s sits of the horizontal arm. This ray then traverses a total of $s + x + s - x = 2s$ sits. According to Pythagoras d-theorem the ray moving in the direction perpendicular to v also moves $2s$ sits (Figure 1.1).

8 The experimental conclusions would be, therefore, the same as in the continuum model, but the explanation of those results would be quite different. In effect, while in the continuum model a length contraction in the direction of v is needed, in the case of a CALM, Pythagoras d-theorem suffices. FitzGerald-Lorentz contraction could then be interpreted as a transformation between the discrete and the continuous models of spacetime. This is the idea we will examine in the next section.

9 In the same way the special theory of relativity is only necessary at velocities close to the speed of light, Pythagoras digital theorem would be necessary only at ultramicroscopic scales close to Planck scale. By way of illustration, take into account that each point of an extremely fine line visually perceived by a human eye, would be composed of a super-millionaire number of sits, so that that line at Planck scale would be a surface whose width would also have a super-millionaire number of sits.

Reinterpreting Lorentz transformation

10 We will begin this section by deriving (in semi formal terms.[1]) Lorentz transformation to emphasize the role of the relativistic factor γ.

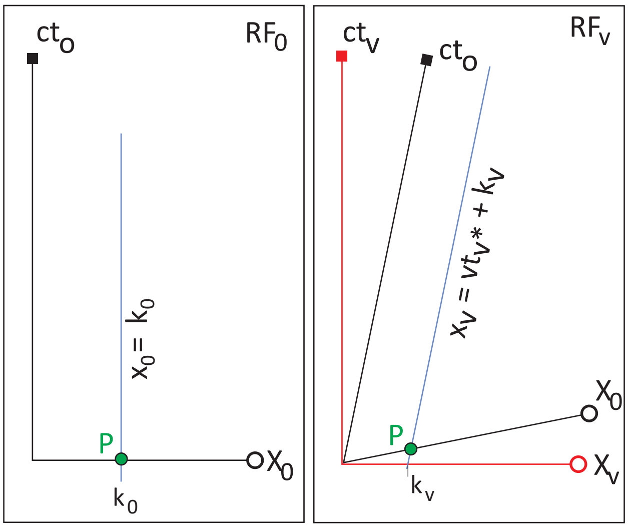

Figure 1.2: Left: world line of a static point in RF_o. Right: world line of the same static point as viewed from RF_v.

11 Let P be a point at rest in its proper inertial reference frame RF_o. The worldline[2] of P in RF_o is the straight line (Figure 1.2, left) whose

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[1] See [1]

[2] The line whose points represent the successive positions of P as time

equation is:

$$x_o = k_o \tag{8}$$

12 Let RF_v be an inertial referential frame that moves relative to RF_o with a velocity v in the direction of X_o. Assume that from the perspective of RF_v the frame RF_o moves from left to right, which evidently means that from the point of view of RF_o it is the frame RF_v that moves from right to left (see Chapter **??** on conventions). The world line of P in RF_v will be therefore (Figure 1.2, right):

$$x_v = vt_v + k_v \tag{9}$$

Or:

$$x_v - vt_v = k_v \tag{10}$$

Dividing (8) by (9) one obtains:

$$\frac{x_o}{x_v - vt_v} = \frac{k_o}{k_v} = k \tag{11}$$

So that:

$$x_o = k(x_v - vt_v) \tag{12}$$

13 We will now make use of the First Principle of Relativity, according to which RF_o and RF_v are totaly equivalent. In consequence, under the same above conditions of 11 the analysis of the world line of a point at rest now in RF_v leads to:

$$x_v = k(x_o + vt_o) \tag{13}$$

Equations (12) and (13) are the new relativistic version of Galileo classical transformation for the X axis (the equations for the Y and Z axes are totaly equivalent).

14 To determine the constant k we make use of the Second Principle of Relativity. In the place of a stationary point, let $x_o = ct_o$ be the X_o

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passes (See the Chapter **??** on conventions).

coordinate of a photon that moves parallel to the X_o axis. Similarly, let $x_v = ct_v$ be the X_v coordinate of a photon that moves parallel to X_v. According to (12) and (13) we can write:

$$ct_o = k(ct_v - vt_v) \tag{14}$$

$$ct_v = k(ct_o + vt_o) \tag{15}$$

Dividing both sides of (14) by t_v:

$$c\frac{t_o}{t_v} = k(c - v) \tag{16}$$

$$\frac{t_v}{t_o} = \frac{c}{k(c - v)} \tag{17}$$

Dividing both sides of (15) by t_o:

$$c\frac{t_v}{t_o} = k(c + v) \tag{18}$$

That is to say:

$$\frac{t_v}{t_o} = \frac{k(c + v)}{c} \tag{19}$$

And taking into account (17):

$$\frac{c}{k(c - v)} = \frac{k(c + v)}{c} \tag{20}$$

$$c^2 = k^2(c^2 - v^2) \tag{21}$$

$$k^2 = \frac{c^2}{c^2 - v^2} \tag{22}$$

$$k = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{23}$$

which is the celebrated relativistic factor γ.

15 By replacing k with $(1 - v^2/c^2)^{-1/2}$ we get Lorentz transformation for the X axis:

$$x_v = \gamma(x_o + vt_o) = \frac{x_o + vt_o}{(1 - v^2/c^2)^{1/2}} \tag{24}$$

$$x_o = \gamma(x_v - vt_v) = \frac{x_v - vt_v}{(1 - v^2/c^2)^{1/2}} \tag{25}$$

Equations for the Y and Z axes have a similar form.

16 The transformation for the time axis requires some elementary algebra. Starting from

$$x_v = \gamma(x_o + vt_o) \tag{26}$$

$$x_o = \gamma(x_v - vt_v) \tag{27}$$

and replacing x_v in (27) by its expression in (26), we immediately obtain:

$$x_o = \gamma(\gamma(x_o + vt_o) - vt_v) \tag{28}$$

$$\frac{x_o}{v\gamma} = \gamma\left(\frac{x_o}{v} + t_o\right) - t_v \tag{29}$$

$$t_v = \gamma\left(\frac{x_o}{v} + t_o\right) - \frac{x_o}{v\gamma} \tag{30}$$

$$= \gamma\left(\frac{x_o}{v} + t_o - \frac{x_o}{v\gamma^2}\right) \tag{31}$$

$$= \gamma \left(t_o + \frac{x_o}{v} \left(1 - \frac{1}{\gamma^2} \right) \right) \tag{32}$$

$$= \gamma \left(t_o + \frac{x_o}{v} \frac{v^2}{c^2} \right) \tag{33}$$

$$= \frac{t_o + \dfrac{x_o v}{c^2}}{\sqrt{1 - v^2/c^2}} = \left(t_o + \frac{x_o v}{c^2} \right) \gamma \tag{34}$$

And similarly:

$$t_o = \frac{t_v - \dfrac{x_v v}{c^2}}{\sqrt{1 - v^2/c^2}} = \left(t_v - \frac{x_v v}{c^2} \right) \gamma \tag{35}$$

This makes clear the role of γ in Lorentz transformation.

17 Assume now that in RF_o a photon traverses the vertical distance L_o from A to B in a time $t_o = L_o/c$ (Figure 1.3, left). From the perspective of RF_v, from which RF_o moves from left to right in the X_o direction, the same photon traverses the distance L_v from A to B, which is now the hypotenuse of the right triangle ABC, in a time t_v (Figure 1.3, right). Evidently, it holds:

$$t_v = \frac{\sqrt{L_o^2 + (vt)^2}}{c} \tag{36}$$

which, after a little algebra, leads to $t_v = \gamma t_o$.

18 As in the case of the special theory relativity, in a CALM framework there is also a unsurmountable maximum velocity (a sit per tit) and each CALM has its own universal laws that always apply in the same form. But γ may be interpreted in a different way. Indeed, according to Pythagoras d-theorem in both frames, RF_o and RF_v, the photon traverses the same number of sits. Thus, to transform discrete into continuum distances (within ultramicroscopic scales close

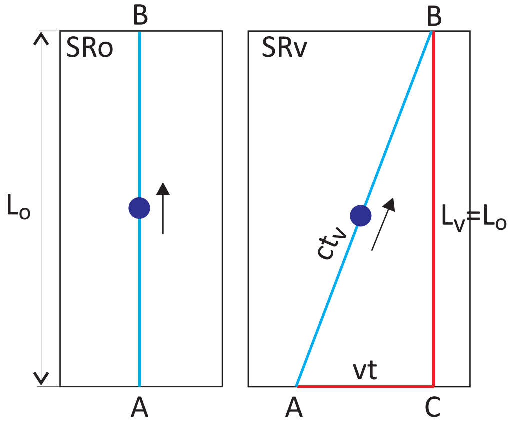

Figure 1.3: Motion of a photon from A to B as seen from RF_o (left) and from RF_v (right). In certain discrete geometries, as our CALM, AB has the same number of sits as BC so that it takes the photon the same time to go from A to B as from C to B.

to Planck scale) we should make use of the ratio (Figure 1.3):

$$\frac{AB}{BC} = \frac{AB}{\sqrt{AB^2 - AC^2}} \tag{37}$$

$$= \frac{1}{\sqrt{1 - AC^2/AB^2}} \tag{38}$$

$$= \frac{1}{\sqrt{1 - (vt_v)^2/(ct_v)^2}} \tag{39}$$

$$= \frac{1}{\sqrt{1 - v^2/c^2}} \tag{40}$$

$$= \gamma \tag{41}$$

The same argument can be applied to the above example 7 of Michelson-Morley-like interferometer, being AB in that case the continuum distance light traverses in the direction perpendicular to v and BC its corresponding discrete distance.

19 Time dilatation, difference in phase synchronization and mass increment with relative motion can also be derived from Lorentz transformation. We should therefore consider the possibility that the special theory of relativity were a consequence of describing a digital world with the analog mathematics of the continuum. If that were the case, the relativistic factor γ, and then Lorentz transformation, could be considered as an operator that transforms digital into continuum distances and times.

20 The above discussion suggests the possibility of two interpretations of the special theory of relativity: the continuum or classical interpretation based on the continuum geometry, and the discrete, digital or discontinuous interpretation based on discrete or discontinuous geometries. In the next chapters will discusses some inconveniences of the continuum interpretation that do not appear in the discrete interpretation.

Bibliography

[1] Max Born, *Einstein’s theory of relativity*, Dover Publications Inc., New York, 1965.