1.-Galileo's stone and Einstein's photon

> [the experiment] will show the stone always lands in the same place, be the ship at rest or moving with any velocity.
>
> Galileo Galilei

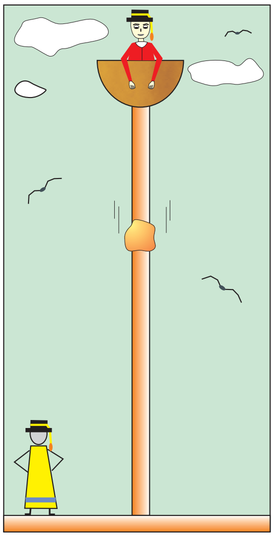

Figure 1.1: For all observers, the falling stone moves parallel to the mast, be it (ship) or not (desk) at rest with respect to the observer.

INTRODUCTION

1 As is well known, the case of Galileo's ship was one of the firsts discussions on the relativity of motion: A stone falling from the top of the mast of a moving ship will, or not, land at the base of the mast? Galileo refers to this experience as actually performed by others but not by him, although he was firmly convinced the stone lands at the base of the mast, as if the ship where at rest in a dock [1].

2 We now know the reasons for which the stone will, in fact, land at the base of the mast, and the reasons for which the observers in the ship observe the stone moves along a vertical trajectory while the observers in the dock observe that trajectory is not vertical: all ship's objects, including the crew of the ship and the stone, move with the same horizontal velocity v and since no force, other than gravity, acts on the falling stone it will continue to move in the horizontal direction with the same velocity v as it moves down (Newton’s first law of mechanics).

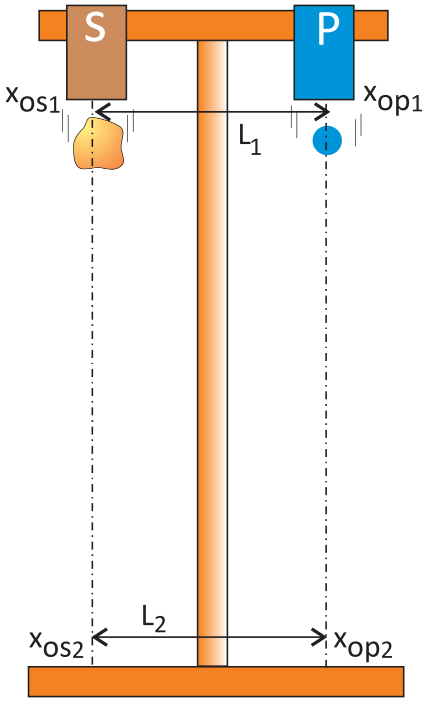

Figure 1.2: Galileo’s mast and Einstein’s photon. Will the stone and the photon move parallel to each other?

3 As we know, the motion of a photon is independent of the motion of its emitting source. Contrarily to Galileo’s stone, it does not inherit the motion of its emitting source. In these conditions, a photon emitted from the ship in the direction parallel to its mast should or not behave as the falling stone? This is the question we will discuss here. A discussion that is intended to prepare the discussion of the next chapter.

Discussion

4 Let’s slightly change the scenario of Galileo discussion on the ship and the faling stone by assuming that at the same instant the stone begins to move down, a photon (Einstein’s photon) is emitted downward in a direction parallel to the ship’s mast and at a certain horizontal distance from the place where the stone is dropped (Figure 1.2).

5 In the next discussion we will examine the trajectories of Galileo’s stone and Einstein’s photon, from the top of the mast where they are released to the points of the ship’s desk where they finally impact. As usual the analysis will be carried out from two different points of views, in the first place from the perspective of the ship (RF_o) and then from that of the dock (RF_v).

6 In what follows we will consider the ship’s mast as the Y_o axis of RF_o and its intersection with the desk as the origin of that frame. From the perspective of this frame the analysis of the stone trajectory is elemental: since only the force of gravity acts on the stone, its dropping trajectory can only be a vertical straight line parallel to the mast.

7 According to (the analog interpretation of) the special theory of relativity, the trajectory of the photon will also be a vertical line parallel to the mast, otherwise the deviation α from the vertical could be used to calculate the absolute velocity u of RF_o. In effect, if y is the length of the mast, x the horizontal displacement of the photon with respect to its initial position, and t_o the time it takes the photon to complete its journey, we would have:

$$\tan\alpha = \frac{x}{y} \tag{1}$$

$$x = ut_o \tag{2}$$

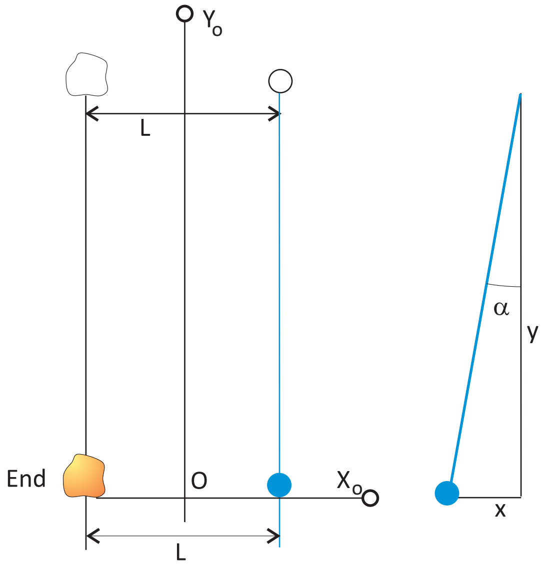

Figure 1.3: In RF_o the stone and the photon follow parallel trajectories (left), otherwise it would be possible to calculate the absolute velocity of RF_o in terms of x (right).

$$u = y \tan \alpha / t_o \tag{3}$$

8 For the observers in the ship (RF_o) it is the dock and the rest of the world (RF_v) that moves with respect to them, and for this relativistic reason they observe the stone and the photon follow parallel trajectories. But, let they reasoning (only for a moment) in non-relativistic terms. These observers know they move with respect to the dock at a velocity they change and control. They also know the reason for which they observe the stone moving down along a vertical trajectory: as they themselves, the stone inherits the velocity of the ship and maintains it while falling down. The photon also follows a vertical trajectory but it does not inherit the velocity of the ship. Then, could those momentarily sceptic observers ask, why the photon follows that trajectory? If the photon does not inherit the motion of the ship why it is observed as inheriting the velocity of the ship? Is the photon trajectory depending upon their way of reasoning?

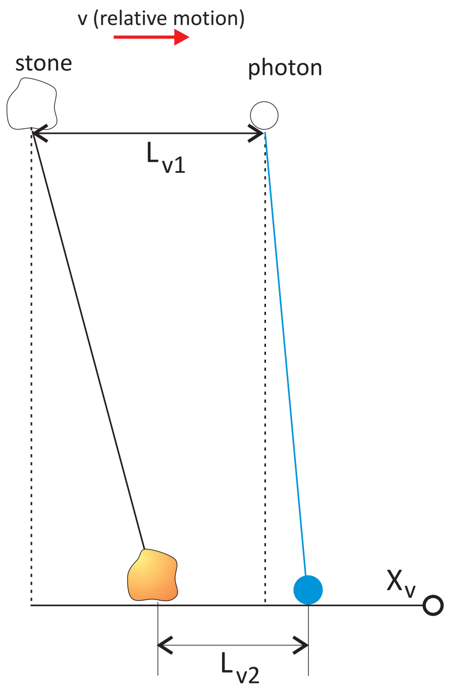

Figure 1.4: From the perspective of the dock (RF_v) the trajectories of the stone and the photon are not parallel.

9 We will prove now the trajectories of the stone and the photon are not parallel in RF_v (dock). Indeed, assume the stone lasts a time t_{vs} to reach the ship's desk. During that time, and with respect to RF_v, it will move a horizontal distance $d_{vs} = vt_{vs}$ in the direction of the relative motion, where v is the velocity of the relative motion (the velocity of the ship as seen from the dock).

10 Assume the photon lasts a time t_{vp} to reach the desk. During that time, and with respect to RF_v, the ship will have moved a horizontal distance $d_{vp} = vt_{vp}$ in the direction of the relative motion. Since the speed of light is greater than the speed of the stone we will have:

$$t_{vs} > t_{vp} \tag{4}$$

Therefore, for RF_v observers, the stone lands at a horizontal distance vt_{vs} from the point it was dropped; while the photon impacts at a horizontal distance vt_{vp} from the point where it was emitted. So we will have:

$$vt_{vs} > vt_{vp} \tag{5}$$

11 In consequence, if L_{v1} and L_{v2} are respectively the initial and final separation between the stone and the photon, then from the perspective of RF_v, it will hold:

$$L_{v2} = L_{v1} - vt_{vs} + vt_{vp} \neq L_{v1} \tag{6}$$

since the ship (RF_o) moves relative to the desk (RF_v) in the direction from the initial position of the stone to the initial position of the photon (Figure 1.4). The inequality $L_{v1} \neq L_{v2}$ proves the trajectories of the stone and the photon are not parallel in RF_v.

12 In consequence, and as in the case of the RF_o observers, the observers in RF_v will see the stone and the photon behave in the same way with respect to the relative motion: their respective trajectories vary with the relative velocity of the ship: the greater its speed the more inclined their trajectories.

13 All observers, whether they are at rest or in relative motion with respect to the ship, will have to conclude that, in spite of the fact that the motion of a photon is independent of the motion of its emitting source, Einstein's photon behaves as any material object mechanically linked to its proper frame, and then as one that inherits the relative motion of its proper frame, as is the case of Galileo's stone.

Bibliography

[1] Galileo Galilei, *Diálogo sobre los dos máximos sistemas del mundo ptolemaico y copernicano*, Círculo de Lectores, Barcelona, 1997.