Part I

Special relativity discussions

1.-FitzGerald-Lorentz contraction

> . . . the question ’Does the Fitzgerald-Lorentz contraction really take place?’ has no single, unequivocal answer from a relativistic point of view.
>
> Anthony P. French

Introduction

1 The visualization of a physical object is accomplished by means of the rays of light that, reflected by the object, reach the corresponding optical system. Due to the finite speed of light and to the different distances light has to traverse from the different parts of the object, not all reflected rays reach the system at the same instant. In consequence, FitzGerald-Lorentz contraction cannot be visually perceived, or photographed, as such a contraction in the direction of the relative motion but as a sort of rotation [9], [11], [8]. Although the appropriate correction for these effects will reveal it to be present.

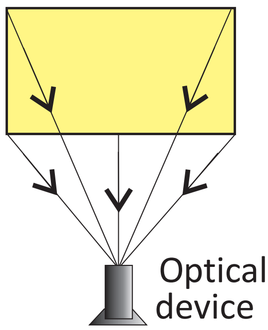

Figure 1.1: Each reflected ray follows a different trajectory towards the optical device.

2 In this book we have assumed the results of all observations and measurements performed by an observer in an inertial reference frame are appropriately corrected in a computer whose screen displays the corrected images. In consequence, all objects observed in relative motion will be considered and represented as actually contracted in the direction of the relative motion, in agreement with FitzGerald-Lorentz contraction.

3 In the previous chapter we had the opportunity to examine the role of the relativistic factor $\gamma = 1/\sqrt{1 - v^2/c^2}$ in Lorentz transformation, the mathematical operator for conversion between the measurements performed in two inertial frames that move relative to each other with a constant velocity. Length contraction, time dilatation, phase difference in synchronization[1] and mass increment with relative motion can, all of them, be derived from that operator.

4 As we will see in the next section, the real or apparent nature of Lorentz contraction remains open to discussion, although the discussions are not very popular. Most of the introductory and university textbooks on the special theory of relativity pay little attention, if any, to it. The real or apparent nature of time dilatation, phase difference in synchronization and mass increment with relative motion are even less discussed in the literature. But be it real or apparent, the nature of these four consequences of relative motion has to be the same, just because all three derive directly from the same Lorentz transformation.

5 The discussion on the real or apparent nature of FitzGerald-Lorentz contraction is further complicated by other external discussions that put into question the very existence of objective realities beyond human observers.

6 Apart from reviewing some opinions on the real or apparent nature of Lorentz transformation, this chapter poses the some relativistic problems derived from FitzGerald-Lorentz contraction, and then from the analog interpretation of Lorentz transformation.

[1]The time difference between two unsynchronized clocks.

FitzGerald-Lorentz contraction

7 In 1887-1892 G.F. FitzGerald [4] and H. A. Lorentz [7] proposed (independently) a *real* length contraction of moving objects in the direction of the motion with respect to the luminiferous aether in order to explain the negative results of Michelson-Morley experiment (see Chapter **??**). According to FitzGerald and Lorentz, the contraction was caused by changes in the intermolecular forces of moving bodies. Since there were no reason to such changes, the proposal was considered as an *ad hoc* hypothesis.

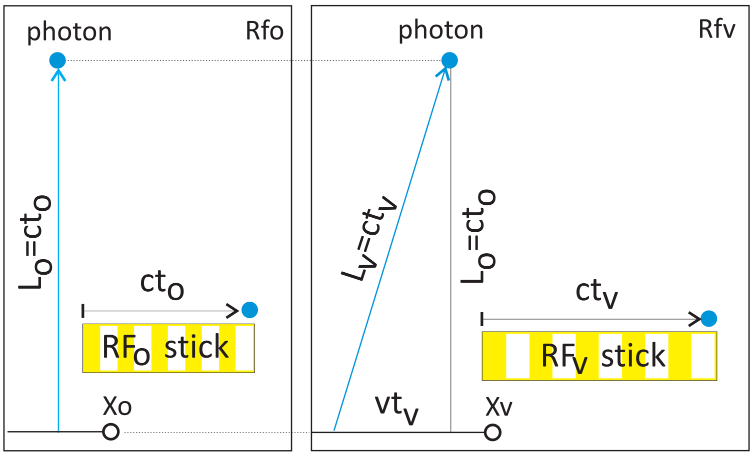

Figure 1.2: Time dilatation and length contraction as a consequence of the principles of the special theory of relativity

8 As it is immediate to see, FitzGerald-Lorentz contraction is a consequence of the principles of the special theory of relativity. In fact, assume in a frame RF_o a photon traverses a vertical distance L_o in a time t_0 at its universal speed c. In other inertial frame RF_v, from which RF_o moves from left to right in the direction of the X_o-axis with a velocity v, the photon traverses the hypothenuse L_v of a right triangle one of whose vertical side is L_o. Since the hypotenuse (in the continuum spacetime) is greater than both catheti and the speed of light is constant, the photon lasts a time t_v greater than t_o to complete its trip (see **??** in the previous chapter):

$$c^2 t_v^2 = c^2 t_o^2 + v^2 t_v^2 \tag{1}$$

$$t_v^2 \left(c^2 - v^2\right) = c^2 t_o^2 \tag{2}$$

$$t_v^2 = \frac{c^2 t_o^2}{c^2 - v^2} \tag{3}$$

$$= \frac{t_o^2}{1 - \frac{v^2}{c^2}} \tag{4}$$

And then:

$$t_v = \gamma t_o > t_o \tag{5}$$

9 Consequently the interval of time (t_o) elapsed in RF_o is less than the one (t_v) elapsed in RF_v. This means that if in RF_o a photon traverses the length L_o of a horizontal metric stick in a time t_o, this time is less than the corresponding time t_v in RF_v, which in turn means that from the RF_v perspective the length of the RF_o-stick has to be seen contracted by a factor γ^{-1} to make it possible that the photon traverses it in a time t_o at its universal speed c.

10 On the other hand, FitzGerald-Lorentz contraction can be immediately derived from Lorentz transformation. In effect, if x_{o1} and x_{o2} are the space coordinates of the two endpoints of a metric stick in RF_o, in the frame RF_v, from which RF_o moves from left to right in the X_o direction with a constant velocity v, the corresponding coordinates x_{v1}, x_{v2} will be such that:

$$x_{o1} = \gamma(x_{v1} - vtv_1) \tag{6}$$

$$x_{o2} = \gamma(x_{v2} - vtv_2) \tag{7}$$

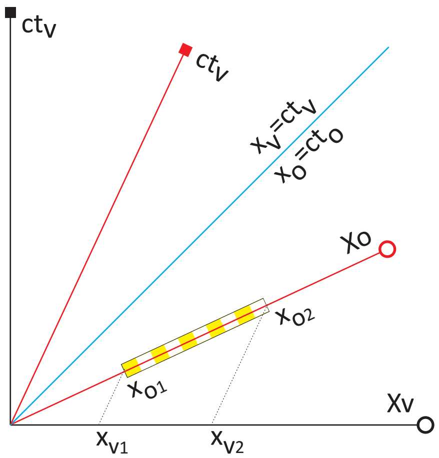

Figure 1.3: Spacetime diagram of a metric stick whose proper space coordinates are x_{o1} and x_{o2}.

And being $t_{v1} = t_{v2}$ in the measurement performed[2] in RF_v, we will have:

$$x_{o2} - x_{o1} = \gamma(x_{v2} - x_{v1}) \tag{8}$$

Or what is the same:

$$x_{v2} - x_{v1} = \gamma^{-1}(x_{o2} - x_{o1}) \tag{9}$$

Real or apparent?

11 We have just seen that FitzGerald-Lorentz contraction is an inevitable consequence of the Principles of Relativity. Now then, is that contraction real[3] or apparent? Most of the authors of books on the special theory of relativity avoid to deal with this ”notorious

[2]To measure a moving stick we would have to measure the position of its endpoints at the same instant, otherwise one side will be displaced with respect to the other and we would get an erroneous measure of the stick.

[3]As some contemporary authors (for instance [1]) claim.

controversy” (as Max Born called it [3]) On this controversy Anthony P. French wrote [5, pp. 113-114]:

> This discussion should make it clear that the question ”Does the FitzGerald-Lorentz contraction really take place?” has no single, unequivocal answer from a relativistic point of view. The whole emphasis is on defining what actual observations we must make if we want to measure the length of some object that may be in motion relative to us. And the prescription is simply that we measure the positions of its ends at the same instant as judged by us. What else could we possibly do? Thus the contraction, when we observe it, is not a property of matter but something inherent in the measuring process.

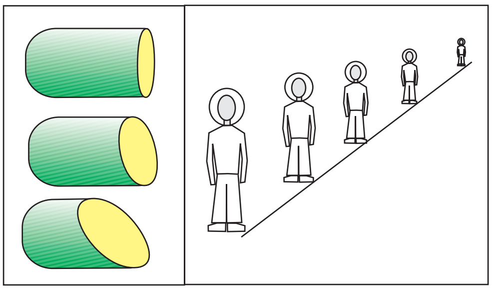

Figure 1.4: Born’s cucumber and Bohm shrinking men.

12 In his now classical book on Einstein’s relativity Max Born wrote [3, pp. 254-55]:

> If we slice a cucumber, the slices will be larger the more obliquely we cut them. It is meaningless to call the sizes of the various oblique slices ”apparent” and call, say, the smallest which we get by slicing perpendicularly to the axis, the ”real” size. In exactly the same way a rod in Einstein’s theory has various lengths according to the point of view of the observer. One of these lengths, the statical or proper length, is the greatest but this does not make it more real than the others.

13 On the same issue, David Bohm wrote [2, Loc. 1253-71 Kindle edition]:

> One may perhaps compare this situation to what happens when two people A and B separate, while still in each other's line of sight. A says that B seems to be getting smaller, while B says that A seems to be getting smaller. Why then does not B say that A seems to be getting larger? The answer is that each is seeing *something different*, i.e. the image of the world on his retina. There is no paradox in the fact that the image of A on B's retina gets smaller at the same time that the image of B on A's retina gets smaller. Similarly, there is no paradox in saying that A will ascribe a contraction to B's ruler, while B ascribes a contraction to A's simply because each is referring to *something different* when he talks about the length of an object.

14 And finally, in a contemporary university textbook of physics we can read [6, p. 1032]:

> Does a moving object really shrink? Reality is based on observations and measurements; if the results are always consistent and if no error can be determined, then what is observed and measured is real. In that sense, the object really does shrink.

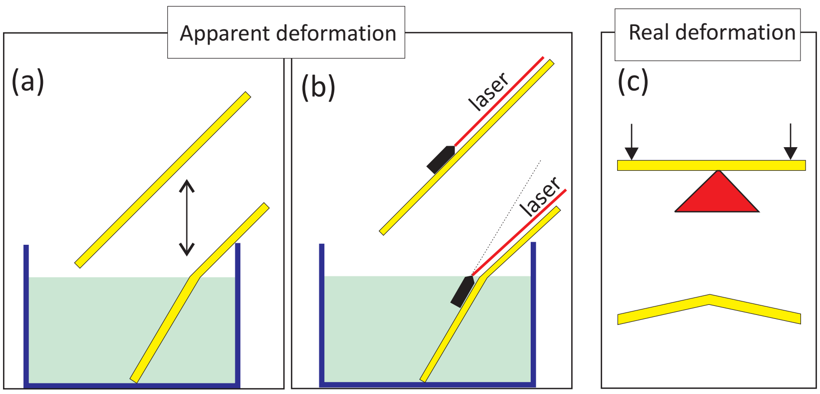

Figure 1.5: Apparent and real deformation. Note that in (b) the laser beam is parallel to the rod even if the laser source is submerged.

15 As a counterpoint, consider now a straight rigid rod partially and obliquely submerged in water. Due to the refraction of light, the rod seems to be bent, but evidently it is not, as can be immediately proved by removing it from the water, or even without removing it (Figure 1.5 (b)). Consider also other rod identical to the first one but mechanically and irreversibly deformed by the application of an appropriate mechanical effort. There is no controversy here: in the first case the deformation is only apparent; in the second the deformation is real.

16 Something changes in the atomic structure of the actually deformed rod, and that change can be experimentally proved, for instance, by means of X-ray diffraction. This is not what happens in the apparently deformed rod, as could also be experimentally proved, for instance by the laser beam method suggested in Figure 1.5. We have then a physically testable asymmetry between an apparent and a real deformation. So it makes sense to speak of real and apparent deformations.

17 Let us now compare FitzGerald-Lorentz contraction with the submerged rod deformation:

- FitzGerald-Lorentz contraction is real in the same sense it is real the bending of the partially submerged rod: both perceptions are not hallucinations of the observers. And both are perfectly explainable in physical terms.

- Both deformations are consequences of two particular ways of observing an object: in relative motion in the first case, and partially submerged in water in the second one.

- If we observe a partially submerged rod we can easily reconstruct its actual shape and size by a simple application of Snell law on the refraction of light. In the same way, if we observe a FitzGerald-Lorentz contracted object we can also reconstruct its real (proper) shape and size by a simple application of Lorentz transformation.

- Both are reversible in the sense that by removing the rod from the water and by decreasing the relative velocity to a null valor both rods recover their original (proper) size and shape.

18 Imagine now that a rod of proper length L_o, placed parallel to the X_o axis of its proper inertial frame, is observed by different observers at different relative velocities parallel to X_o. Imagine also each observer measures the apparent deformations of the same rod partially submerged in different fluids and at different inclinations. Imagine, finally, that once finished their experimental jobs, all observers meet in the same inertial frame and discuss their observations. Will their conclusions on the apparent or real nature of Lorentz contraction be the same as their conclusions on the apparent or real deformation of a partially submerged rod?

19 Unfortunate expressions like 'motion modifies the length of the moving objects' are more frequent than expected in the literature on the special theory of relativity. They are indeed unfortunate because 'motion modifies' makes one think in absolute motion, which is obviously inconsistent with the Principles of Relativity. In addition it is taken for granted that the deformation is real. Although less spectacular, it would be more accurate and less confusing to say that an object, when observed in relative motion, is (or seems to be) contracted in the direction of the relative motion. And by consequence, the same should be said on time dilatation, difference in phase synchronization and mass increment with relative motion.

20 The idea, on the other hand, that concrete realities cannot exist without human observers makes it impossible the existence of human observers because the concrete history of life from which they have evolved could not have been posible without human observers. That said, and recalling Ockham's razor, should not we use the world "real" only in the cases as the rod mechanically deformed and the word "apparent" in the cases as the partially submerged rod? In this sense, is Fitzgerald-Lorentz contraction real as the mechanically deformed rod or apparent as the submerged one? Or perhaps it is real (or apparent) in other sense. And if this were the case, in which sense?

21 As with length contraction, inertial dilatation of time, phase difference in synchronization and increment of mass with relative motion are also direct consequences of Lorentz analog transformation. Consequently, the status on their apparent or real nature is in the same situation as in the case of FitzGerald-Lorentz contraction.

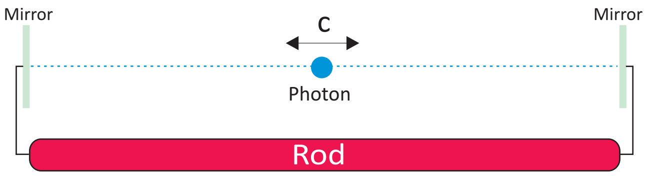

Figure 1.6: The *apparent?* length of a rod seen in relative motion can be corrected only if we also correct the corresponding inertial time dilatation and phase difference in synchronization.

22 Imagine a photon that is continually reflected by two mirrors placed at the ends of an horizontal rod (Figure 1.6). An observer in relative motion with respect to the rod in a direction parallel to it, will see the rod contracted by a factor γ^{-1}. If he corrects FitzGerald-Lorentz contraction he will also have to correct the time dilatation and the phase difference in synchronization for a correct calculation of the photon speed.

23 As noted above, there exists a notable asymmetry in the discussions on the real or apparent nature of FitzGerald-Lorentz contractions with respect to the same discussion (the real or apparent nature) in the cases of time dilatation, phase difference in synchronization or mass increment with relative motion (time dilatation and mass increment may be caused by other physical phenomena different from relative motion). Only the first one is ’relatively’ discussed in the literature. But be it real or apparent, the nature of those four consequences of relative motion should be the same.

24 Let us end this section by paraphrasing the following quote from a university text ([10, p. 42]):

> (Original) I need to warn you about language. I have said that a rod with length L_o as observed from its own frame has a shorter length L_v as observed from another frame. Often this result is

stated as "A rod with length L_o as observed from its own frame appears to have a shorter length L_v as observed from another frame." This statement is true: the rod appears to have shorter length L_v because it does have shorter length L_v. Using the term "appears" gives the false impression that, when the rod is observed from a frame in which it moves, the rod really is of length L_o and only appears to be of length L_v. No. As observed from a frame in which it moves, the rod really does have the shorter length LV.

(Paraphrase) I need to warn you about language. I have said that a straight rod is bent when observed partially submerged in water. Often this result is stated as "A straight rod partially submerged in water appears bent" This statement is true: the rod appears bent because it is bent. Using the term "appears" gives the false impression that, when the rod is observed partially submerged, the rod really is straight and only appears to be bent. No. As observed partially submerged, the rod is really bent.

CHANGING SHAPES

25 Consider now a square steel plate P whose side has a proper length L_o and assume it is oriented in such a way that one of its sides is parallel to the X_o axis of its proper reference frame RFo. As expected, after a clockwise rotation of 45º around its center, P continues to be a square plate in RFo.

26 However, in other inertial frame RFv from which RF_o moves in the X_o direction, before the rotation P is seen as a rectangle whose sides are L_o and $L_o\gamma^{-1}$. After a rotation of 45º the shape of P changes to a rhombus whose side is $L(1+\gamma^{-2})^{1/2}/2$.

27 That a single rotation suffices to change the shape of a metal plate goes against a lot of mechanical, physicochemical and crystallographical laws. On the other hand, and according to the first principle of the special theory of relativity, physical laws are the same in all reference frames. This incongruence should be a sufficient reason

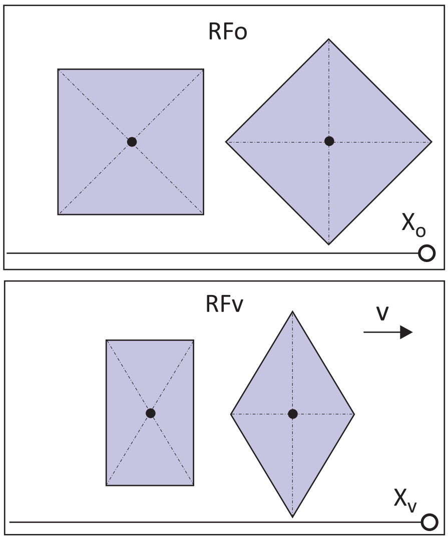

Figure 1.7: According to the observers in *RFv*, a single rotation suffices to change the shape of a metal plate made of the strongest steel.

for RF_v observers conclude their observations are distorted by relative motion *in the same way* the observation of a partially submerged rod is distorted by the refraction of light.

28 The right angled structure depicted in Figure 1.8 poses a similar problem: a single rotation of 90º degrees suffices to alternatively contract one of its arm by a factor γ^{-1} and, at the same time, expanded the other by a factor γ.

29 Let finally B be a bubble of a certain fluid F_2 within other fluid F_1 with which it is in hydrostatic equilibrium. In its proper frame RF_o, the bubble has an spherical shape due to the fact that the hydrostatic pressure is the same in all directions. In RF_v that moves relative to RF_o in the X_o direction, the bubble has an elipsoidal shape due to Lorentz contraction in the direction of the relative motion. Will the observers of RF_v explain the bubble shape in terms of a real or of an apparent deformation?

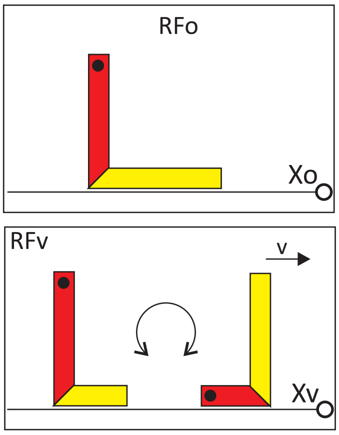

Figure 1.8: Deformation without deforming efforts. By rotating the structure, its two arms contract and expand *alternatively* without any force acts on them.

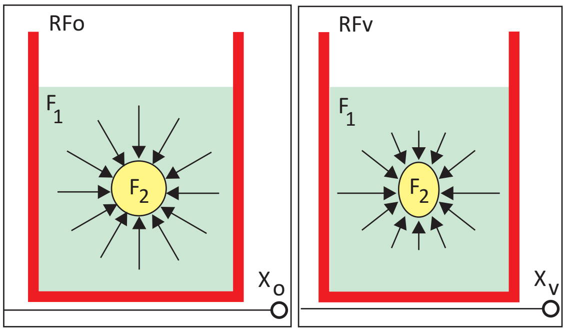

Figure 1.9: A bubble in hydrostatic equilibrium as seen from its proper frame RF_o (left) and from other reference frame RF_v that moves relative to RF_o with a velocity v in the X_o direction.

THE SLIDING PULLEY

30 A more intriguing consequence of Fitzgerald-Lorentz contraction is the next case of the sliding pulley, in which several laws of mechanics are (apparently?) violated.

31 RF_o is the proper frame of the mechanical device schematically

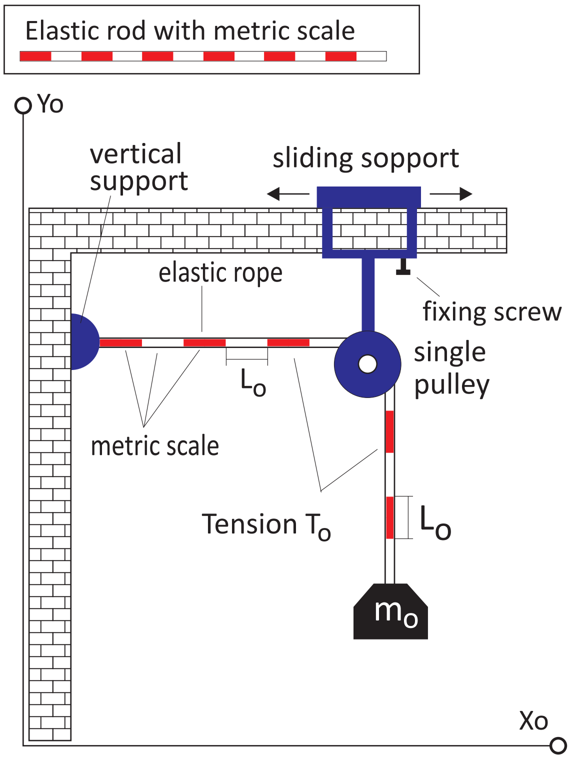

Figure 1.10: According to the laws of mechanics, at equilibrium all red and white marks of the elastic cord can only have the same length.

depicted in Figure 1.10 in which a load of rest mass m_o hangs from an elastic cord whose left end is attached to a fixed vertical support. The cord runs around a pulley which in turn is attached to an horizontal sliding support whose position can be set with the corresponding fixing screw. In addition, a metric scale, which consists of a succession of red and white marks of equal length each, is printed on the elastic cord (Figure 1.10). At equilibrium, the mechanical tension can only be constant along the whole cord, in consequence all marks of its metric scale will have the same length, whether they are in the horizontal or in the vertical section of the cord. A conclusion that, being an immediate consequence of the laws of mechanics, should hold in all reference frames.

32 Let L_o be the proper length of the metric scale marks, and let us assume the X_o axis of RF_o is parallel to the horizontal section of the cord. By sliding its horizontal support, the pulley moves towards the left or towards the right, changing then the length of both the horizontal and the vertical section of the cord; or what is the same, changing the number of the horizontal and vertical marks. As expected, the length of all vertical and horizontal metric marks remains constant and equal to L_o.

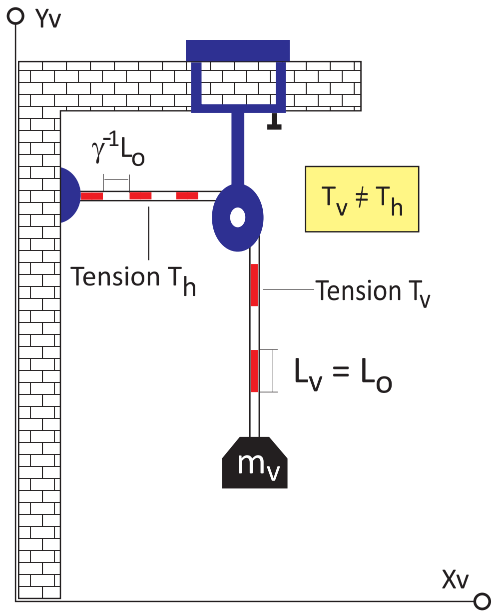

Figure 1.11: In RF_v, from which RF_o moves from left to right in the X_v direction, the length of the horizontal metric marks of the cord are contracted by a factor γ^{-1} while the vertical marks retain its proper length L_o.

33 RF_v is an inertial reference frame that coincides with RF_o at instant $t = 0$ and from which RF_o moves from left to right, in the X_o, direction with a velocity v. In consequence, and according to

Lorentz transformation, in RF_v the length of the vertical marks of the metric scale is L_o, while the horizontal marks have a contracted length $\gamma^{-1}L_o$, independently of the number of both marks.

34 Let n_h be the number of horizontal marks and N the total number of marks. Obviously the number of vertical marks will be $N - n_h$. In RF_o the length L_{oC} of the cord at equilibrium with the pending mass is always the same:

$$L_{oC} = n_h L_o + (N - n_h)L_o \tag{10}$$

$$= L_o(n_h + N - n_h) = NL_o \tag{11}$$

However, in RF_v the length L_{vc} of the cord is variable, depending on the number n_h of horizontal marks:

$$L_{vC} = n_h\gamma^{-1}L_o + (N - n_h)L_o \tag{12}$$

$$= L_o(n_h\gamma^{-1} + N - n_h) \tag{13}$$

$$= L_o(N - n_h(1 - \gamma^{-1})) \tag{14}$$

So, as the number n_h of horizontal marks increases the cord length L_{vC} decreases. Or in other word, as the pulley moves to the right the length of the cord decreases, and as it moves to the left it increases. Obviously these length changes are mechanically impossible in the cord's proper frame RF_o.

35 By sliding the pulley towards the left or towards the right we can make any part of the rope to move from the horizontal to the vertical position and viceversa. When changing, its mechanical tension also changes, being greater when its position is vertical (Figure 1.12). These tension changes are also mechanically impossible in RF_o.

36 As we have just seen, the mechanical situation observed from RF_v is incompatible with the known physical laws. On the one hand the tension of the cord has to be the same along its entire length and,

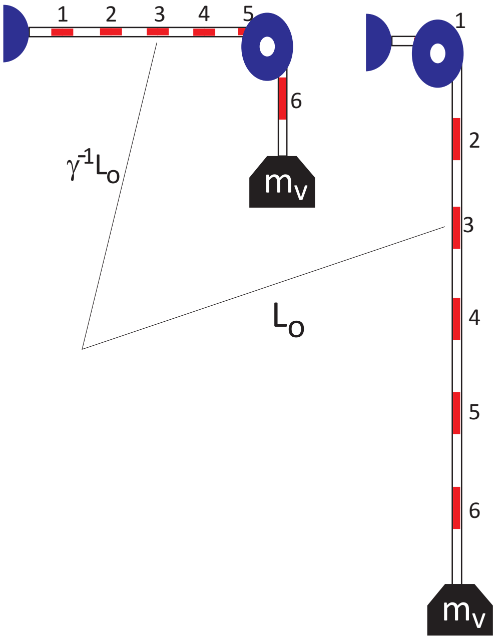

Figure 1.12: The horizontal mechanical tension of any part of the cord changes when it changes its position from the horizontal to the vertical and viceversa.

therefore, the length of all its metric marks has also to be the same, be them horizontal or vertical. On the other, the length of the cord at equilibrium with the pending mass has to be always the same, it cannot change wit the number of horizontal marks, i.e. with the horizontal displacement of the pulley.

37 The observers in RF_v would have to conclude the Lorentz contraction they observe is only apparent, as is apparent the bending of the rod partially submerged in water. Otherwise, they would have to explain the strange mechanical behaviour of the elastic cord: they would have to explain by Lorentz transformation (the source of all special relativity effects) how it is possible for an elastic cord not to have the same mechanical tension along its entire length.

38 By symmetry, if the contraction of lengths by relative motion were apparent, so would be for time dilatation, phase difference in synchronization and mass increment by relative motion (as we will see in Chapter **??** on Hooke's law there is other non-relativistic source of mass increment derived from the equivalence between mass and energy).

39 In short, all observers in relative motion with respect to RF_o should consider the possibility their observations and measurements are distorted by relative motion in such a way they cannot get conclusion physically acceptable on what happens in RF_o. In these sense, only the observers in RF_o may conclude their observations and measurements agree with what is expected from physical laws.

40 There is a third alternative: that length contractions, time dilatation, phase difference in synchronization and mass increment with relative motion (all of them derived from Lorentz transformation) are but consequences of interpreting the discrete reality by indiscrete mathematics. Obviously, this would be the simplest of the three alternatives.

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