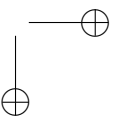
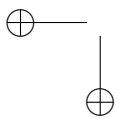
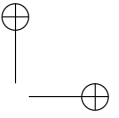
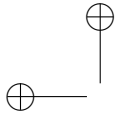


Part I

Towards a
Digital Paradigm



1.-Digital versus analog

... how hard it is for even the most adventurous thinkers to give up beliefs that have been held for millennia.

Lee Smolin

THE ANALOG PARADIGM

1 Modern science is founded on an old Presocratic paradigm, the analog paradigm. A conception of reality based on the idea of the continuum, on the idea that something that can be divided can, in fact, be infinitely divided.

2 Being severely limited by its biological functionality, our sensorial perception of reality is clearly continuous. A world spatially pixelated at the cellular scale, let alone at the Planck scale, would be sensorially perceived as continuous. With respect to time, the perception of continuity is even more deceptive: sixteen fixed images per second suffice to give a sense of continuity. For these reasons it seems reasonable to think that sensation of continuity is behind the old tradition of the analog paradigm, and then behind the continuum-based conceptions of physical entities as matter, energy, charge, space or time.

3 The continuum, in turn, is not an independent notion. It is a formal consequence of the actual infinity hypothesis, other Presocratic invention. The Axiom of Infinity subsumes the hypothesis of

4 — Digital versus analog

the actual infinity and legitimises the idea of the continuum, the assumption of the infinite divisibility.

4 More than twenty-seven centuries have been needed to discover that things are not what they seem to be; that we only perceive a ridiculous fraction of reality. For these reasons and for the lack of both scientific experience and scientific culture at least until the Scientific Revolution, it is very unlikely that Presocratic conceptions of reality pointed in the right direction. Presocratics inaugurated the scientific age, but there still remains a long way to go.

5 The world of science, on the other hand, seems excessively self-reverent, and that excess of self-reverence (that becomes religious devotion in the cases of the great gurus of science), could be making it more difficult than needed to make progress in the Knowledge of reality.

6 At the beginning of the twentieth century (1900), Max Planck had the idea of replacing an integral with a discrete finite summation to solve the ultraviolet catastrophe. The consequences are well known: the birth of quantum mechanics, the most successful science ever developed by man. But Quantum mechanics, the science of discrete entities, is made of indiscrete mathematics. Contemporary mathematics, including mathematical physics, continue to be essentially based on the idea of the continuum. The special theory of relativity, for instance, is a physico-mathematical theory on the continuum spacetime.

7 We have always presumed that if something can be divided then it can be divided infinitely many times. Although the things we have been able to divide never divide in that way. Ordinary matter, elementary particles, electromagnetic energy, electric and non electric charges, are all of them of a discrete nature, with indivisible minima. Only space and time remain as possible infinitely divisible entities. And the prevailing idea is, at least in formal terms, that they are infinitely divisible. The result is what we call the continuum spacetime: between any two points (instants) infinitely many other points (instants) do exist. Exactly 2^{\aleph_0} , a non denumerable infinitude.

8 Let us recall what is assumed when one assumes the hypothesis of the actual infinity (Axiom of Infinity). Consider the list of natural numbers in their natural order of precedence: 1, 2, 3, ... According to the hypothesis of the actual infinity that list exists as a *complete totality*, i.e as a totality that contains, all at once, all natural numbers. The ordered list of natural numbers does exist as a *complete* totality in spite of the fact that no last number *completes* the list. Modern infinitists defend that *all* natural numbers could be counted, even in a finite time by performing the following supertask:¹

Count each number n at the precise instant t_n , the n -th instant of an ω -ordered sequence $\langle t_n \rangle_{n \in \mathbb{N}} = t_1, t_2, t_3, \dots$ of instants within the finite interval (t_a, t_b) , being t_b the limit of $\langle t_n \rangle_{n \in \mathbb{N}}$. Recall that in an ω -ordered sequence (as the sequence 1, 2, 3, ...) there is a first element and each element has an immediate successor and an immediate predecessor, except the first one that has no predecessor.

9 The alternative to the actual infinity hypothesis is the hypothesis of the potential infinity, which rejects the the possibility of counting all natural numbers and then the existence of *complete* infinite totalities. From this perspective, the natural numbers result from the endless process of counting: it is always possible to count numbers greater than any given number. But it is impossible to complete the process of counting, so that the complete list of all natural numbers makes no sense.

10 Nothing in the history of science can be compared with the fecundity of the actual infinity as a source of paradoxes. At the end of the XIX century, one of those paradoxes served to lay the foundations of modern transfinite mathematics [11]. Ever since, infinitism maintains an hegemonic position, almost free of criticism, in contemporary mathematics. But, as could not be otherwise, the actual infinity can also be questioned.²

¹On the concept of supertask see for instance [15] and the section on Thomson's lamp in Chapter .

²Take a glance at [Infinity at stake, http://interciencia.es/infinity.html](http://interciencia.es/infinity.html).

6 — Digital versus analog

11 In addition, the actual infinity does not get along with reality. Nothing that we can observe, measure or manipulate is infinite. Nothing. And when infinity appears in physics equations, physicists are forced to remove it from them because of the unsolvable problems it invariably leads to. A removal that usually requires a lot of hard work, as in the cases of renormalization in particle physics.³ And when quantum mechanics and general relativity try to meet each other the actual infinity also appears making it impossible the encounter [13, p. 73]:

The continuum assumption on space and time seems then to be the root of our problems in quantum gravity.

12 It is worth noting the fact that physicists never question the formal consistency of the actual infinity hypothesis, as if that consistency had been demonstrated. Nothing further from the truth: the actual infinity is only a hypothesis. Authors of the intellectual stature of Brouwer, Poincaré or Wittgenstein, among others, rejected it. But, evidently, the only way to liberate experimental sciences from the burden of the actual infinity would be to prove its formal inconsistency.

13 The inconsistency of the actual infinity would have enormous consequences in both theoretical and experimental sciences, particularly in physics. One of them would surely be the replacing of our current analog paradigm by a digital conception of reality. That change of perspective would be so intense that it could lead to a second scientific revolution, even more intense than the first one.

14 In fact, the paradigm shift is underway: matter, energy, electric charge, . . . are all of them discontinuous. But the most decisive step has not yet been given: to prove the discrete nature of space and time and to develop the appropriate discrete mathematics (the existing ones can be considered rather as approximative versions of the continuum based mathematics).

³[6], [9], [7], [8].

CELL AUTOMAT LIKE MODELS (CALMS)

15 After two centuries of experimental analysis, all continuums, except the spacetime continuum, had to be ruled out by the evidence of facts. The world seems to be essentially discontinuous, discrete, digital.

16 The discrete nature of spacetime has also been proposed in different areas of physics⁴, although the proposed models continue to be developed within the continuum-mathematics framework. Cell automata, on the other hand, have also been proposed to found a new science [22], although so far the proposal is almost empty of content.

17 Cell automata like models (CALM thereafter) have some interesting characteristics that might be useful to start the construction of a new digital paradigm:

1. Space is composed of indivisible units.
2. Each space unit (sit) may have a sufficiently large number of different states.
3. Each state is defined by a set of variables.
4. The state of each sit is maintained for an indivisible time unit (tit).
5. The state of all sits are synchronically updated at each successive tit according to certain laws.
6. All sits vibrate (change their states) in a synchronized manner.

18 Think for a moment in our current analog paradigm: at each ‘instant’ the state of each (spacial or material) ‘point’ also changes or remain unchanged according to certain laws, and the change is a consequence of its precedent state. In addition, and according to quantum nonlocality, changes at the quantum scale can also be synchronic. Thus, from this functional point of view, both models could be more similar than expected. In any case, CALMs seem to be more simple and less extravagant than some other current cosmological theories.

⁴[10], [8], [21], [5], [16], [3], [17], [19], [4], [12], [13], [1].

8 — Digital versus analog

19 Although this work does not intend to propose a particular and detailed CALM, some additional characteristics and definitions should be proposed, at least in informal terms, to make it possible the next discussions. The following ones will suffice:

Motion: *An objet moves if it occupies different contiguous sits at different successive tits (tits’ successiveness has not to be immediate).*

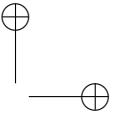
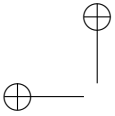
Velocity: *The velocity s/t of an object is the number (s) of sits the object traverses during a certain number (t) of immediately successive tits, being always $s \leq t$.*

20 From the kinematical perspective of a CALM, what is relevant is the number of sits traversed in a given number of tits, not the particular shape and extension of each sit. And not only from a kinematical perspective, perhaps the same relevance applies to geometry. A discrete space is not a grided continuum space⁵ but a space composed of discrete indivisible units, so it is the number of such unities what really *counts*. Contrarily, what counts in the continuum model is the continue extension, infinitely dense, of things. If real numbers are necessary to represent those infinitely dense extensions, rational numbers suffice in the case of the discrete world of CALMs.

21 It is worth noting that in a discrete space nothing can be less than a sit. Thus, a sit cannot be made of other components, it cannot have internal structure since the corresponding structural elements would necessarily have a size less than a sit. A sit is the indivisible and elementary unit of space and then one that defines the minimum size of any other elementary component of any other physical object.

22 In a cell automaton an object is a set of cells that maintains its (static or dynamic) shape. That is to say, a set of cells whose states evolved in a certain common way. Objects are then defined in terms of common ways of evolving, where evolving means changes in the state of the corresponding cells along the successive automaton’s

⁵As is the case of, for instance, of [20].



generations. In a discrete spacetime, could physical objects be also defined in terms of common ways of evolving of sets of sits along the successive tits?

PYTHAGORAS DIGITAL THEOREM

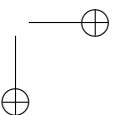
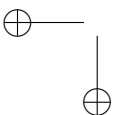
23 In the next chapter we shall have the opportunity to reinterpret Lorentz transformation as an operator that transforms continuum into discontinuum geometries, and viceversa. Evidently, in those conditions the special theory of relativity could also be reinterpreted in the same sense. Length contractions time dilations and difference in phase synchronization, that seem not to have a physical explanation, could be the consequence of expressing the discrete nature of reality through an indiscrete mathematical instrumental. The key of that interpretation will be the next discrete version of Pythagoras theorem.

24 Since each sit updates its state at each successive tit, changes can propagate at a maximum velocity of one sit per tit. Thus, as we have just noted, from a “kinematical” point of view what seems relevant is the number of sits “traversed” in a certain number of tits. For this reason we shall now suggest a numerical version of Pythagoras theorem exclusively based on the number of sits of the sides of a right triangle.

25 Without going into further details, assume the triangles depicted in Figure 1.1 are right-angled in the geometry of a bidimensional CALM. A simple recount⁶ shows that in each triangle the hypotenuse and the greater of the legs have the same number of sits. Thus, and assuming the number of sits is the relevant magnitude, we can state the following digital version of Pythagoras theorem (Pythagoras d-theorem from now on):

Pythagoras d-Theorem. *-In the discrete geometry of a CALM,*

⁶Although we shall not develop it here, a formal proof of Pythagoras digital theorem is also possible.



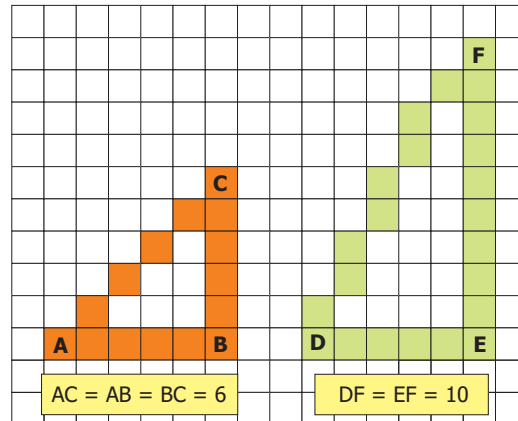


Figure 1.1: An elementary example of CALM. Note that in each right angled triangle the number of hypotenuse's sits is the same as the number of sits of the greater leg. And note also the ultramicroscopic scale, think of a sit as a Planck volume ($\simeq 4.22 \times 10^{-96}mm^3$).

the hypotenuse of a right angled triangle has the same number of sits as the greater of its leg.

Obviously, this digital version of Pythagoras theorem would be necessary only at ultramicroscopic scales close to Planck scale. As we shall see immediately, Pythagoras d-theorem could also be used to test the discrete nature of spacetime.

26 The following conceptual experiment is only a theoretical idea surely inapplicable in the real world. Of course, we ignore if CALMs are appropriate models to represent the physical world. We are simply suggesting the convenience to begin to explore the possibilities of a digital paradigm to explain the physical world in discrete terms. It is, therefore, a simple theoretical speculation that illustrates a new way of conceiving experiments.⁷

⁷While writing these pages an experiment that is being developed near Chicago will attempt to prove, among other things, the discrete nature of spacetime [14].

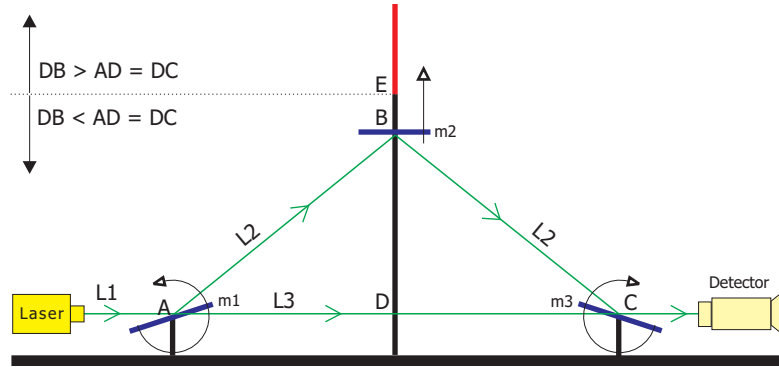


Figure 1.2: Schematic depiction of an interferometer to test the digital nature of spacetime.

27 By way of illustration, consider the interferometer depicted in Figure 1.2. The semi-silvered mirror $m1$ divides the laser beam $L1$ into the beam $L2$ (that moves towards the mirror $m2$) and the beam $L3$ (that moves towards the mirror $m3$). $L2$ is reflected by mirror $m2$ and then moves to $m3$ where it is reflected in the same direction as $L3$, with which it interferes. The interference is then observed with the appropriate detector. Mirrors $m1$ and $m3$ can synchronically rotate in opposite directions while mirror $m2$ can move up and down so that $L2$ meets $L3$ in the same place of $m3$. The legs AD and DC of the right triangles ADB and DBC are always of the same length, but the common leg DB changes as the mirror $m2$ moves up and down.

28 According to Pythagoras d-theorem, while B is below E it will hold: $AB = AD = BC = DC$. Thus, we could move $m2$ up and down and observe the same interference pattern whenever B remains below E . In these conditions, in fact, $L2$ and $L3$ traverse the same number of sits. On the contrary, if B moves above E it will hold $AB = DB > AD$; $BC = DB > DC$ and a different pattern of interference should be observed because in this case the number of sits $L2$ traverses increases as DB increases, while the number of sits traversed by $L3$ remains constant. In short, the discrete nature of spacetime would be experimentally proven if two interference patterns were observed: the one when $m2$ moves below E ; the other when it does it above E .

WHY A DIGITAL PARADIGM?

29 The change of paradigm we are proposing here is indeed a change in the conceptions of space and time. And the main reason to propose such a change is the possible inconsistency of the actual infinity hypothesis on which the idea of the continuum spacetime is based. But as we will see in this section, there are some other additional reasons. These additional reasons should suffice to analyze the convenience of the proposal, even if the actual infinity hypothesis were consistent, what, in my opinion, is not the case.

30 With respect to the structure of spacetime, the main difference between the analog and the digital models is the lack of successiveness of points and instants in the continuum spacetime of the analog model: being densely ordered, no point in the continuum spacetime has an immediate successor. Each point of the continuum spacetime has an uncountable infinitude of successors, but none of them succeeds it in the same way the number four succeeds the number three in the ordered list of natural numbers.

31 In CALMs, on the contrary, successiveness is a well defined characteristic of both space and time: in any spacial direction each sit has an immediate successor (its adjacent neighbouring in that direction); and each tit is immediately succeeded by other tit of identical duration, so that no time elapses between two successive tits.

32 As Appendix ?? shows, spacetime successiveness is necessary to solve the problem of change. And, although physicists seem to have forgotten that fundamental problem, nature will never be explained in consistent terms without having previously resolved the problem of change.

33 In addition, points and instants seem to be undefinable notions (at least in the platonic scenario of the analog paradigm) and then of no physical significance anyway.

34 It seems reasonable that science consider all alternatives in order to explain the world. Although it seems also reasonable to expect a lot of difficulties to change the course of a well established current of

thought, as is the case of the analog paradigm, which has been the exclusive paradigm in the last twenty seven centuries.

35 As we know, computers can produce an unlimited variety of virtual worlds, but all of them have to be discrete, ‘pixelated’. Computers cannot produce nor simulate virtual worlds of analog structure (the simulations will always be truncated). This is a really significant restriction. On the one hand, it could explain why we had not been confronted with the analog versus digital dilemma before the development of computing sciences. On the other it poses the question of whether only discrete worlds, virtual or not, can really exist.

36 In any case, it could be interesting to end this section by enumerating some of the most relevant (at least apparently) advantages of CALMs:

1. The actual infinity is not necessary.⁸
2. The problem of change is finally resolved (Appendix ??).
3. The infinite regression of arguments [2], an unsurmountable barrier in all fields of human Knowledge, could also be finally explained (although not surmounted).
4. Contrarily to points and instants, sits and tits do have physical significance.
5. CALMs are much more simple than the continuum model: while between any two points of the continuum spacetime uncountably many other (exactly 2^{\aleph_0}) points exist, the number of sits in the whole visible universe would be finite ($\approx 2.66 \times 10^{185}$ if they were cubes of a Planck’s volume).
6. In the analog models of nature, extension and shape loss their physical meaning at the elementary particle level, that meaning could be found within the discrete spacetime of CALMs.
7. Since each of the variables defining the state of a sit is updated at each successive tit, sits can vibrate in multiple forms. A few number of sits should suffice to define elementary objects as superstrings or elementary particles.

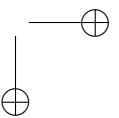
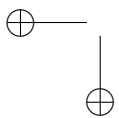
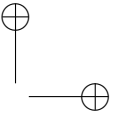
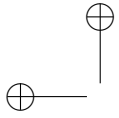
⁸The actual infinity could be inconsistent (Chapter 1).

8. From the superstring theory perspective, if a sit, or a few number of them, suffices to define a large number of different vibrations, ultramicroscopic extra dimensions could be unnecessary.
9. Nothing can last a time less than one tit. The maximum speed in a CALM is therefore one sit per tit. That could be the speed of light in the vacuum, although not necessarily. Particles faster than light could exist in a CALM if the speed of light is less than one sit per tit.
10. The speed of light, or of any other physical object, does not depend upon the relative motion of the observer that perform the measurement: whatsoever be the number of sits an object traverse in a given number of tits, both the number and the particular sits the object traverses will be the same for all observers.
11. Motion, and then physical laws, have not to be referred to abstract reference frames but to the actual fabric of sits of a CALM.
12. The special theory of relativity could be reinterpreted making it unnecessary some of its inconveniences. This book deals with some of them.
13. Synchronicity, one of the most pervasive characteristic of nature (from atoms to organisms), and the main source of its self-organizing capacity [18], can be naturally explained in terms of CALMs.
14. The entanglement of space and time finds a natural explanation within the logic of CALMs functioning.
15. The flow of time and its irreversible arrow, enigmatic from a continuum spacetime perspective, is naturally explained in CALM terms.
16. The slippery concept of *now* could also be easily explained in CALM terms.
17. The incessant quantum activity of 'free space' could be better explained in CALM terms than it is in the continuum spacetime.
18. Quantum nonlocality could be a natural consequence of CALMs synchronized way of functioning.

19. Bohm potential, wave pilot and some other related quantum concepts could also be easily interpreted in a CALM framework.
20. The place and role of information in the physical world could also be explained in terms of CALMs.
21. Although quantum mechanics seems to be complete, the continuum spacetime it makes use of could be inappropriate. If that were the case, realist theories (and then the common sense in science) could recover its lost validity.
22. Observers, instruments and observed objects would form part, all of them, of the same CALM. Their current mutual interactions would determine their irreversible future.
23. General relativity and quantum mechanics would have a new (discrete) opportunity to meet each other [1].

Although, it is reasonable to suspect, the most interesting consequences of discrete paradigms as CALMs are still to be discovered.

37 In the following chapters we will make use of the special theory of relativity to test the convenience of replacing the analog with the discrete paradigm. We will see that, in fact, some relativistic effects as length contractions, time dilatations and differences in phase synchronization by relative motions, could be indicating the discrete nature of space and time. Those enigmatic effects could be the consequence of ignoring the digital nature of the world, of forcing an indiscrete explanation of what simply is a discrete world. Perhaps we should let Ockham razor to begin doing its work.



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