THE PROBLEM OF CHANGE

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Abstract. Change is not only the most pervasive feature of nature, it is also the most elusive question ever posed by man.[1] So elusive that it could be inconsistent, as it has been claimed at least from Presocratic times. Evidently, if that were the case the task of explaining nature in consistent terms would be impossible. This paper defines the concept of canonical change, poses the problem of its consistency from the perspective of its duration and proves that only in a discrete spacetime could it find a consistent solution.

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1. Definitions

1-1. For the sake of simplicity, and in order to avoid unnecessary discussions, we will discuss here the problem of causal changes in physical macroscopic objects. So, if O is one of those macroscopic objects, we will say O changes causally from the state S_a to the state S_b, symbolically:

$$S_a \mapsto S_b \tag{1}$$

if there exist a set of physical laws $\mathcal{L}$ such that, under the same conditions C and as a consequence of those laws, the state of O is S_a at instant t and S_b at an ulterior instant t'. In symbols:

$$\mathcal{L}(S_a, C) = S_b \tag{2}$$

1-2. The change $S_a \mapsto S_b$ can be direct, without intermediate states, in whose case it will be referred to as *canonical* change. It can also be the result of an ordered sequence of canonical changes:

$$\{S_i\} : S_a \mapsto S_1 \mapsto S_2 \mapsto S_3 \mapsto \ldots \mapsto S_b \tag{3}$$

Notice that every element S_n of $\{S_i\}$ has to have an immediate predecessor S_{n-1} (except the first of them) so that S_n can be causally derived from S_{n-1}:

$$\forall S_n \in \{S_i\} : \; \mathcal{L}(S_{n-1}, C_{n-1}) = S_n \tag{4}$$

1-3. Assume that $S_a \mapsto S_b$ takes place through a densely ordered sequence of causal changes $[S_a, S_b]$. Being $[S_a, S_b]$ densely ordered, there is no element S_α within (S_a, S_b) such that:

$$\mathcal{L}(S_\alpha, C_\alpha) = S_b \tag{5}$$

because if that where the case S_α would be the immediate predecessor of S_a and then the sequence would not be densely ordered. The same argument could be applied to any element in $[S_a, S_b]$. Some infinitists claim that although S_b does not result as a consequence of a causal canonical change from an immediately precedent state, it results from the completion of an infinite sequence of precedent changes. We will next examine this possibility.

1-4. Assume S_b results from the completion of an infinite sequence of changes. The state of our object O will be S_a at a certain instant t_a, and S_b at other certain instant t_b. In those conditions, let $f(t)$, for each t in $[t_a, t_b]$, be the number of those changes that have still to be performed at instant t in order to reach S_b. It is immediate that $f(t)$ can only take two values: either $2^{\aleph_o}$ or 0. In effect, if $f(t)$ could take a finite value n then there would exist the impossible lasts n changes of a densely ordered sequence of changes. For the same reason, if n were denumerably infinite then there would exist an impossible denumerable number of lasts changes in a densely ordered sequence of changes.

1-5. According to 1-4, $f(t)$ defines a dichotomy: the number of changes to be performed at each instant t in $[t_a, t_b]$ to reach S_b, can only be either $2^{\aleph_o}$ or 0. In consequence, there is no instant within $[t_a, t_b]$ at which only a finite number of changes remain to be performed in order to reach S_b. Or in other words, that number has to change *directly* from $2^{\aleph_o}$ to 0, which is possible unless uncountably many changes take place instantaneously. In fact, assume the transition from $2^{\aleph_0}$ to 0 in the number of changes to be performed takes a certain amount of time τ, being τ any positive real number, including $t_b - t_a$. At any t within the interval $(0, \tau)$ the number of changes to be performed can only be $2^{\aleph_0}$ (dichotomy of $f(t)$) and then the transition in question takes a time less than τ, for any positive real number τ. So, that transition can only take a time null. As we will see, instantaneous changes are not possible in the continuum spacetime.

1-6. If the sequence of changes from S_a to S_b were α-ordered, being α any denumerable transfinite ordinal, there would be at least one state without immediate predecessor,[2] but there would also be infinitely many states with both an immediate predecessor and an immediate successor, and then an infinitude of canonical changes.

[2] That would be the case if the sequence were ω-ordered, being ω the less transfinite ordinal.

2. The problem of change

2-1. Consider any canonical change of the above sequence $\{S_i\}$ defined by (3), for instance the first of them $S_a \mapsto S_1$. We will begin by proving that change has to be instantaneous, i.e. of a null duration in the continuum spacetime. In fact, assume its duration is $\tau > 0$, being τ any positive real number. For every t in the real interval $(0, \tau)$, the state of our object O will be either S_a or S_1. If it were S_a then the change would not yet have begun and its duration would be equal or less than $\tau - t$ in the place of τ. If it were S_1 then the change would have already finished and its duration would be equal or less than t in the place of τ. But O has to be in one of those two states because $S_a \mapsto S_1$ is a canonical change. Consequently, and taking into account that τ is any positive real number, we conclude that no positive real number can be the duration of the canonical change $S_a \mapsto S_1$. It has to be, therefore, instantaneous.

2-2. We will prove now that instantaneous causal changes are impossible in a continuum spacetime. As we will see, the reason for that impossibility is that no instant t of a densely ordered sequence of instants has an immediate successor t^s (or predecessor t^p) such that no time elapses from t to t^s (or from t^p to t). Assume the instantaneous canonical change $S_a \mapsto S_1$ takes place at a certain instant t_{a1}. The change could be instantaneous if the state of O is S_a at t_{a1} and S_1 at an hypothetical immediate successor t^s_{a1} of t_{a1}, so that the time elapsed between t_{a1} and t^s_{a1} is null; or, alternatively, if O is in S_1 at t_{a1} and in S_a at an hypothetical immediate predecessor t^p_{a1} of t_{a1}, being also null the time elapsed between t^p_{a1} and t_{a1}. But in the continuum spacetime both alternatives are impossible because t_{a1} has neither immediate predecessor nor immediate successor. We must therefore conclude the canonical change $S_a \mapsto S_1$ cannot be instantaneous in a continuum spacetime.

2-7. We have just proved that if a change takes place through a densely ordered sequence of changes then infinitely many changes take place instantaneously (1-5); we have also proved that canonical changes can only be instantaneous (2-1) and that instantaneous changes are impossible in the continuum spacetime (2-2). We must conclude change is impossible in the continuum spacetime. In the next section, we will analyze its possibilities in a discrete spacetime.

3. A discrete model: cell automata

3-1. Cell automata like models (CALM) provide a new interesting perspective to analyze the way the universe seems to works. It provides a discrete spacetime framework from which it could be possible a new analysis of some of the apparently unsolvable problems and paradoxical situations in modern physics, particularly those related to quantum mechanics and relativity. As we will see in the next short discussion, twenty seven centuries after it was posed, the old problem of change finds its first consistent solution in that discrete framework.

3-2. In CALMs, space is exclusively composed of indivisible minimal units we will term qsits (quantum space units). Time is also composed of a sequence of successive indivisible units, qtits (quantum time uits) in our notation. Each qsit may exhibit different states defined, each one of them, by a certain set of variables. The state of all qsits change simultaneously in each qtit in accordance with the laws driving the evolution of the automaton.

3-3. Let u, v, c, $\dots z$ be the set of variables defining the states of the qsits of a certain CALM A. Let us represent the n-th state of each qsit s_i by $s_i(u_n, v_n, \dots z_n)$, where u_n, v_n $\dots$ are the particular values of the state variables at the n-th qtit. Let finally $\mathcal{L}$ be the set of laws that control the evolution of the automaton. $\mathcal{L}$ determine the way each qsit changes from a qtit to the next one taking into account the state of that qsit as well as the state of any other interacting qsit, which may include all qsits of A.

3-4. The automaton 'engine' changes the state of every qsit at each qtit and maintains it just along one qtit. Thus, if A_n represents the state of the automaton at the n-th qtit we can write:

$$\mathcal{L}(A_n, C_n) = A_{n+1} \tag{6}$$

$$\mathcal{L}(A_{n+1}, C_{n+1}) = A_{n+2} \tag{7}$$

$$\mathcal{L}(A_{n+2}, C_{n+2}) = A_{n+3} \tag{8}$$

$$\dots$$

and for each particular qsit s_i:

$$\mathcal{L}(s_i(u_n, v_n, \dots, z_n), C_n) = s_i(u_{n+1}, v_{n+1}, \dots, z_{n+1}) \tag{9}$$

$$\mathcal{L}(s_i(u_{n+1}, v_{n+1}, \dots, z_{n+1}), C_{n+1}) = s_i(u_{n+2}, v_{n+2}, \dots, z_{n+2}) \tag{10}$$

$$\mathcal{L}(s_i(u_{n+2}, v_{n+2}, \dots, z_{n+2}), C_{n+2}) = s_i(u_{n+3}, v_{n+3}, \dots, z_{n+3}) \tag{11}$$

$$\dots$$

3-5. Being both space and time discrete, each qtit t_n has an immediate predecessor t_{n-1} and an immediate successor t_{n+1}, so that no other qtit exists neither between t_{n-1} and t_n nor between t_n and t_{n+1}. Or in other words: no time elapses between any two successive qtits. This simple characteristic of CALMs suffices to solve the problem of change: discrete spacetime allows instantaneous changes: the state A_n at qtit t_n changes to A_{n+1} in the next qtit t_{n+1}. And this is possible because the state of each qsit is redefined at each qtit and maintained at least for a qtit.

References

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